12

Diffraction of Light

12.1 DIFFRACTION

Light is known to travel in straight lines. This is a direct inference from the formation of shadows of opaque obstacle. However, it was discovered that with small sources, the shadow of a small object is much larger than that given by geometrical construction and is surrounded by fringes. This can be explained only if one assumes that light travels in the form of waves and bends round the corners of an obstacle. This phenomenon of deviation of light from rectilinear propagation and bending round the corners of an obstacle in known as diffraction. This is an important phenomenon exhibited by waves. In the case of sound waves for which wave-length is sufficiently large diffraction phenomenon can be easily observed. But in the case of light, the wavelength is extremely small and a very careful setting and closer observation is required. The size of the obstacle should be of dimensions comparable with the wave length of light. Careful experiments reveal that there is encroachment of light in the geometrical shadow region of opaque obstacles. Further the intensity of illumination outside the geometrical shadow region is not uniform but shows variation.

The essential difference between interference and diffraction of light is that, in interference the resultant intensity at a point is the resultant of superposition of two wavefronts coming from two coherent sources, whereas in the diffraction phenomenon the resultant intensity at a point is due to superposition of wavelets from two parts of a single wavefront.

12.2 CLASSIFICATION OF DIFFRACTION

The phenomena of diffraction of light is divided into the following two classes depending upon the position of source and the place of observation with respect to the diffracting obstacle.

12.3 FRESNEL'S CLASS OF DIFFRACTION

In this class of diffraction the source of light, or the screen or both are usually at finite distance from the obstacle. The wavefronts employed are spherical or cylindrical. They are treated by construction of half-period zones. The diffraction patterns obtained under this class are very faint, because in these the wavelets reaching any point of the screen from different parts of the exposed wavefront are all in different phases and produce only a feeble resultant. On the screen we get a pattern which is of the shape of the obstacle with some modifications due to diffraction. Further the pattern is formed in a plane which is not focally conjugate to the plane in which the source lies.

12.4 FRAUNHOFFER CLASS OF DIFFRACTION

In this class of diffraction the source and the screen are effectively at infinite distance from the obstacle. The incident light is diffracted in various directions and that diffracted in a particular direction is focussed on a screen by means of a convex lens. The illumination at the screen is greater if the phases of these parallel rays happen to agree. It is not necessary to employ a plane wavefront, we may even employ spherical or cylindrical wavefronts to obtain this class of diffraction. In that case the essential condition is that the pattern must be observed in a plane which is conjugate to the plane in which the source lies. In this class of diffraction the shape of the source is reproduced in the pattern as modified by the diffracting aperture. The diffracting aperture or obstacle do not come in the diffraction pattern.

12.5 FRESNEL'S ASSUMPTIONS

According to Fresnel, the resultant effect at an external point due to a wavefront will depend on the factors discussed below:

In Fig. 12.1, *S* is a point source of monochromatic light and *MN* is a small aperture, *XY* is the screen and *SO* is perpendicular to *XY*. *MCN* is the incident spherical wavefront due to the point source *S*. To obtain the resultant effect at a point *P* on the screen, Fresnel assumed that (1) a wavefront can be divided into a large number of strips or zones called Fresnel's zones of small area and the resultant effect at any point will depend on the combined effect of all the secondary waves emanating from the various zones; (2) the effect at a point due to any particular zone will depend on the distance of the point from the zone; (3) the effect at P will also depend on the obliquity of the point with reference to the zone under consideration, e.g., due to the part of the wavefront at *C*, the effect will be maximum at *O* and decreases with increasing obliquity. It is maximum in a direction radially outwards from *C* and it decreases in the opposite direction. The effect at a point due to obliquity factor is proportional to $(1 + \cos\theta)$ where $\angle PCO = \theta$. Considering an elementary wavefront at *C*, the effect is maximum at *O* because $\theta = 0$ and $\cos\theta = 1$, Similarly, in a direction tangential to the primary wavefront at *C* (along *CQ*) the resultant effect is one half of that along *CO* because $\theta = 90°$ and $\cos 90° = 0$. In this direction *CS*, the resultant effect is zero since $\theta = 180°$ and $\cos 180° = -1$ and $1 + \cos 180° = 1 - 1 = 0$. This property of the secondary waves eliminates one of the difficulties experienced with the simpler form of Huygens principle viz., that if the secondary waves spread out in all directions from each point on the primary wavefront, they should give a wave travelling forward as well as backward as the amplitude at the rear of the wave is zero there will evidently be no back wave.

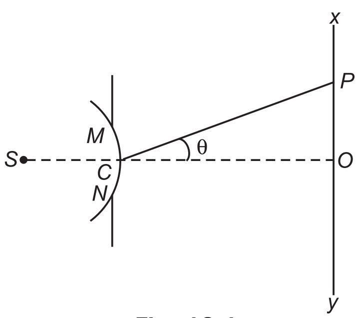


Fig. 12.1

12.6 RECTILINEAR PROPAGATION OF LIGHT

ABCD is a plane wavefront perpendicular to the plane of the paper Fig. 12.2(*a*) and *P* is an external point at a distance *b* perpendicular to *ABCD*. To find the resultant intensity at *P* due to the wavefront *ABCD*, Fresnel's method consists in dividing the wavefront into a number of

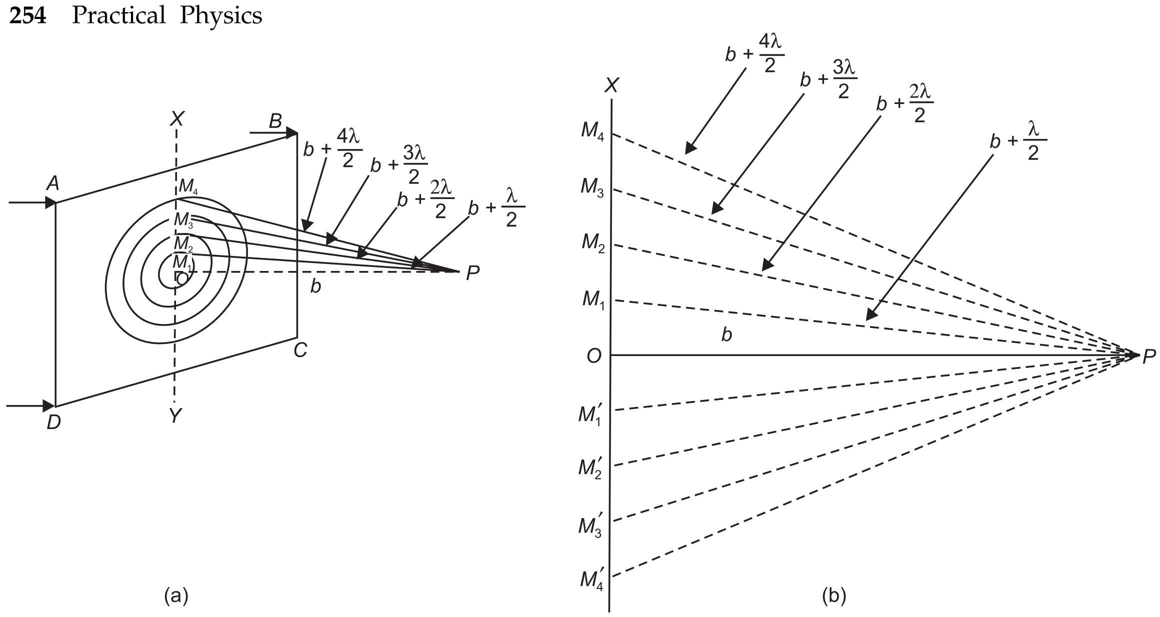
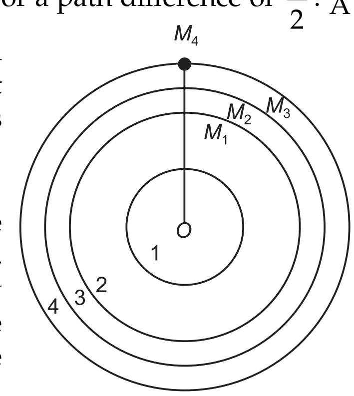


Fig. 12.2

half period elements or zones called Fresnel's zones and to find the effect of all the zones at the point *P*.

With *P* as centre and radii equal to $b+\frac{\lambda}{2}$, $b+\frac{2\lambda}{2}$, $b+\frac{3\lambda}{2}$ etc. construct spheres which will cut out circular areas of radii OM_1, OM_2, OM_3, etc., on the wavefront. These circular zones are called half-period zones or half period elements. Each zone differs from its neighbour by a phase difference of π or a path difference of $\frac{\lambda}{2}$. Thus the secondary waves starting from the point *O* and M_1 and reaching *P* will have a phase difference of π or a path difference of $\frac{\lambda}{2}$. A Fresnel half period zone with respect to an actual point *P* is a thin annular zone (or a thin rectangular strip) of the primary wavefront in which the secondary waves from any two corresponding points of neighbouring zones differ in path by $\frac{\lambda}{2}$.

In Fig. 12.2(*c*), *O* is the pole of the wavefront *XY* with reference to the extrnal point *P*. *OP* is perpendicular to *XY*. In Fig. 12.2(*c*) 1, 2, 3 etc. are the half period zones constructed on the primary wavefront *XY*. OM_1 is the radius of the first zone. OM_2 is the radius of the second zone and so on *P* is the point at which the resultant intensity has to be calculated.



(c)

Fig. 12.2

$$OP = b,\ OM_1 = r_1\ OM_2 = r_2,\ OM_3 = r_3 \text{ etc.}$$

and $$M_1P = b + \frac{\lambda}{2},\ M_2B = b + \frac{2\lambda}{2},\ M_3P = \frac{3\lambda}{2} \text{ etc.}$$

area of the first half period zone is

$$\pi OM_1^2 = \pi\,[M_1P^2 - OP^2]$$

$$= \left[\left(b+\frac{\lambda}{2}\right)^2 - b^2\right] = \pi\left[b\lambda + \frac{\lambda^2}{2}\right] = \pi b\lambda \text{ approximately} \quad ...(i)$$

(As λ is small, λ^2 term is negligible).
The radius of the first half period zone is

$$r_1 = OM_1 = \sqrt{b\lambda} \quad ...(ii)$$

The radium of the second half period zone is

$$OM_2 = \left[M_2P^2 - OP^2\right]^{\frac{1}{2}} = \left[(b+\lambda)^2 - b^2\right]^{\frac{1}{2}} = \sqrt{2b\lambda} \text{ approximately}$$

The area of the second half period zone is

$$= \pi\left[OM_2^2 - OM_1^2\right] = \pi\left[2b\lambda - b\lambda\right] = \pi b\lambda$$

Thus, the area of each half period zone is equal to $\pi b\lambda$. Also the radii of the 1st, 2nd, 3rd etc. half period zones are $\sqrt{1b\lambda}, \sqrt{2b\lambda}, \sqrt{3b\lambda}$ etc. Therefore, the radii are proportional to the square roots of the natural numbers. However, it should be remembered that the area of the zones are not constant but are dependent on (i) λ the wave length of light and (ii) b, the distance of the point from the wavefront. The area of the zone increases with increase in the wavelength of light and with increase in the distance of the point P from the wavefront.

The effect at a point P will depend on (i) the distance of P from the wavefront, (ii) the area of the zone, and (iii) the obliquity factor.

Here, the area of each zone is the same. The secondary waves reaching the point P are continuously out of phase and is phase with reference to the central or the first half period zone. Let m_1, m_2, m_3 etc. represent the amplitudes of vibration of the ether particles at P due to secondary waves from the 1st, 2nd, 3rd etc. half period zones. As we consider the zones outwards from O, the obliquity increases and hence the quantities m_1, m_2, m_3 etc. are of continuously decreasing order. Thus, m_1 is slightly greater than m_2; m_2 is slightly greater than m_3 and so on. Due to the phase difference of π between any two consecutive zones, if the displacement of the ether particles due to odd numbered zones is in the positive direction, then due to the even numbered zones the displacement will be in the negative direction at the same instant. As the amplitude are of gradually decreasing magnitude, the amplitude of vibration at P due to any zone can be approximately taken as the mean of the amplitudes due to the zones preceding and succeeding it. e.g.

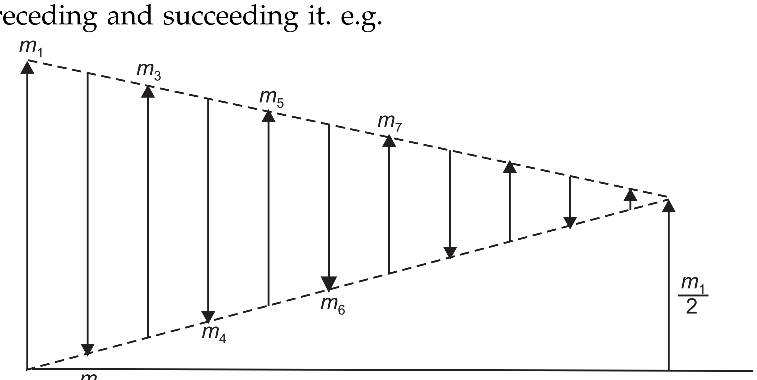


Fig. 12.3

$$m_2 = \frac{m_1 + m_3}{2}$$

The resultant amplitude at *P at* any instant is given by

$$A = m_1 - m_2 + m_3 - m_4 ... + m_n \text{ if } n \text{ is odd.}$$

(If n is even, the last quantity is $-m_n$)

$$\therefore \quad A = \frac{m_1}{2} + \left[\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right] + \left[\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right] + \cdots$$

But $$m_2 = \frac{m_1}{2} + \frac{m_3}{2} \text{ and } m_4 = \frac{m_3}{2} + \frac{m_5}{2}$$

$$\therefore \quad A = \frac{m_1}{2} + \frac{m_n}{2} \cdots \text{ if n is odd}$$

and $$A = \frac{m_1}{2} + \frac{m_{n-1}}{2} - m_n \cdots \text{ if } n \text{ is even.}$$

If the whole wavefront *ABCD* is unobstructed the number of half period zones that can be constructed with reference to the point *P* is infinite i.e., $n \to \infty$. As the amplitudes are of gradually diminishing order, m_n and m_{n-1} tend to be zero.

Therefore, the resultant amplitude at *P* due to the whole wavefront = $A = \frac{m_1}{2}$. The intensity at a point is proportional to the square of the amplitude

$$\therefore \quad I \propto \frac{m_1^2}{4}$$

Thus, the intensity at *P* is only one-fourth of that due to the first half period zone alone Here, only half the area of the first half period zone is effective in producing the illumination at the point *P*. A small obstacle of the size of half the area of the first half period zone placed at *O* will screen the effect of the whole wavefront and the intensity at *P* due to the rest of the wavefront will be zero. While considering the rectilinear propagation of light the size of the obstacle used is far greater than the area of the first half period zone and hence the bending effect of light round corners (diffraction effects) cannot be noticed. In the case of sound waves, the wavelengths are far greater than the wavelength of light and hence the area of the first half period zone for a plane wavefront in sound is very large. If the effect of sound at a point beyond as obstacle is to be shadowed, an obstacle of very large size has to be used to get no sound effect. If the size of the obstacle placed in the path of light is comparable to the wavelength of light, then it is possible to observe illumination in the region of the geometrical shadow also. Thus, rectilinear propagation of light is only approximately true.

12.7 ZONE PLATE

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. It can be designed so as to cut off light due to the even numbered zones or that due to the odd numbered zones. The correctness of Fresnel's method is dividing a wavefront into half period zones can be verified with its help.

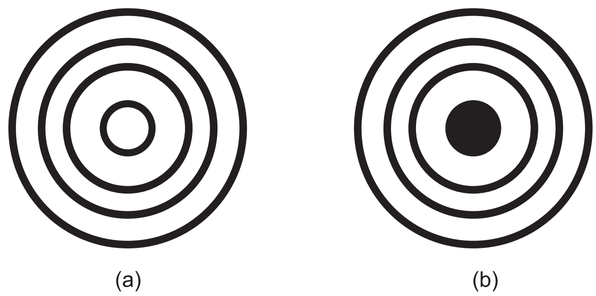


Fig. 12.4

To Construct a zone plate, concentric circles are drawn on white paper such that the radii are proportional to the square roots of the natural numbers. The odd numbered zones (i.e., 1st, 3rd, 5th etc) are covered with black ink and a reduced photograph is taken. The drawing appears as shown in Fig. 12.4(b) the negative of the photograph will be as shown in Fig 12.4(a). In the developed negative, the odd zones are transparent to incident light and the even zones will cut off light.

If such a plate is held perpendicular to an incident beam of light and a screen is moved on the other side to get the image, it will be observed that maximum brightness is possible at some position of the screen say b cm from the zone plate. XO is the upper half of the incident plane wavefront. P is the point at which the light intensity is to be considered. The distance of the point P from the wavefront is b. OM_1 ($= r_1$), OM_2 ($= r_2$) etc. are the radii of the zones,

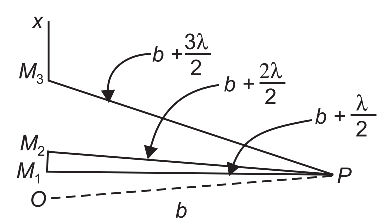


Fig. 12.5

$$r_1 = \sqrt{b\lambda} \text{ and } r_2 = \sqrt{2b\lambda}$$

Where λ is the wavelength of light

$$r_n = \sqrt{n\,b\lambda} \text{ or } b = \frac{r_n^2}{n\lambda}$$

If the source is at a large distance from the zone plate, a bright spot will be obtained at P. As the distance of the source is large, the incident wavefront can be taken as a plane one with respect to the small area of the zone plate. The even numbered zones cut off the light and hence the resultant amplitude at $P = A = m_1 + m_3 + \ldots$ etc.

In this case the focal length of the zone plate f_n is given by

$$f_n = b = \frac{r_n^2}{n\lambda} \qquad \because r_n^2 = bn\lambda$$

Thus, a zone plate has different foci for different wavelengths, the radius of the n^{th} zone increases with increasing value of λ. It is very interesting to note that as the even numbered zones are opaque, the intensity at P is much greater than that when the whole wavefront is exposed to the point P.

In the first case the resultant amplitude is given by

$$A = m_1 + m_3 + m_5 + \cdots m_n \ (n \text{ is odd})$$

When the whole wavefront is unobstructed the amplitude is given by

$$A = m_1 - m_2 + m_3 - m_4 + \cdots + m_n$$

$$= \frac{m_1}{2}. \text{ (if } n \text{ is very large and } n \text{ is odd).}$$

If a parallel beam of white light is incident on the zone plate, different colours came to focus at different points along the line *OP*. Thus, the function of a zone plate is similar to that of a convex (converging) lens and a formula connecting the distance of the object and image paints can be obtained for a zone plate also.

12.8 ACTION OF A ZONE PLATE FOR AN INCIDENT SPHERICAL WAVEFRONT

Let *XY* represent the section of the zone plate perpendicular to the plane of the paper. *S* is a point source of light, *P* is the position of the screen for a bright image, *a* is the distance of the source from the zone plate and *b* is the distance of the screen from the plate. OM_1, OM_2, OM_3, (r_1, r_2, r_3) etc. are the radii of the 1st, 2nd, 3rd etc half period zones. The position of the screen is such that from one zone to the next there is an increasing path difference of $\frac{\lambda}{2}$.

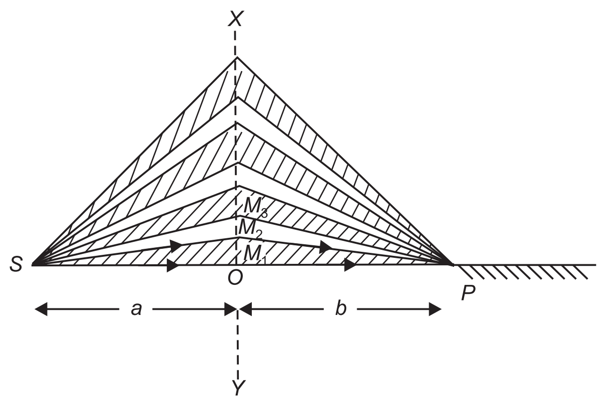


Fig. 12.6

Thus, from Fig. 12.6,

$$SO + OP = a + b$$

$$SM_1 + M_1P = a + b + \frac{\lambda}{2} \qquad \text{...(i)}$$

$$SM_2 + M_2P = a + b + \frac{2\lambda}{2} \text{ and so on}$$

From the $\Delta\, SM_1O$,

$$SM_1 = \left(SO^2 + OM_1^2\right)^{\frac{1}{2}} = \left(a^2 + r_1^2\right)^{\frac{1}{2}}$$

Similarly from the $\Delta\, OM_1P$

$$M_1P = \left(OP^2 + OM_1^2\right)^{\frac{1}{2}}$$

$$= \left(b^2 + r_1^2\right)^{\frac{1}{2}}$$

Substituting the values of SM_1 and M_1P in eqn. (i)

$$\left(a^2 + r_1^2\right)^{\frac{1}{2}} + \left(b^2 + r_1^2\right)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$a\left(1 + \frac{r_1^2}{a^2}\right)^{\frac{1}{2}} + b\left(1 + \frac{r_1^2}{b^2}\right)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b} = a + b + \frac{\lambda}{2}$$

$$\frac{r_1^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{\lambda}{2}$$

$$r_1^2\left(\frac{1}{a} + \frac{1}{b}\right) = \lambda$$

Similarly for r_n, i.e., the radius of the nth zone, the relation can be written as

$$r_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$

Applying the sign convention

or
$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n} \quad \text{...(ii)}$$

$$f_n = \frac{r_n^2}{n\lambda}$$

Equation (ii) is similar to the equation $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$ in the case of lenses with a and b as the object and image distances and f_n the focal length. Thus, a zone plate acts as a converging lens. A zone plate has a number of foci which depend on the number of zones used as well as the wavelength of light employed.

12.9 OBJECT

Determination of the diameter of a wire by diffraction.

Apparatus used: Optical bench with accessories, sodium lamp, and Ramsden's eye- piece with micrometer screw.

Formula used: $d = \frac{D\lambda}{W}$ where W is fringe width, D is the diatance between the pin and the screen, d is diameter of pin.

Theory: Consider a cylindrical-wave front WW' Fig. 12.7 of wavelength coming from a slit S, normal to the plane of paper, falling on a narrow pin having a finite width or diameter $AB = d$, the sides of the pin being parallel to the length of slit. On the screen MM', the geometrical shadow region will be represented by PP'.

The effect of inserting AB in the path of light is to screen of a few of the half period elements. At a point K outside the geometrical shadow, the effect is given by the sum of resultant effects of the half period elements in the upper half of the wave front above the pole O', with respect to point K and those elements which lie between O' and edge of obstacle. If the obstacle is large the effect of half period elements of the lower half of the wave front WW' which are not obstructed by wire, may be neglected. This is because of the fact that these half period elements will be of higher order.

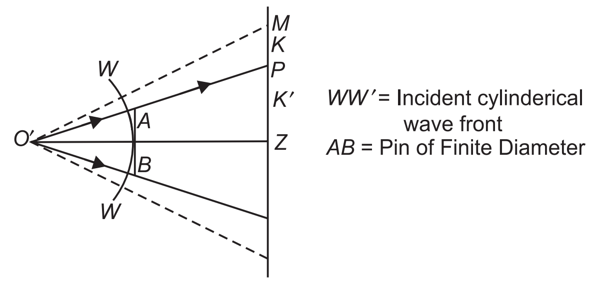


Fig. 12.7

If $O'A$ contains as even number of half period strips the point K will be comparatively dark. If $O'A$ contains odd number of half period strips, the point K will be bright.

The diffraction patterns of the 2 sides of the geometrical shadow are thus similar to the diffraction pattern outside the geometrical shadow of straight edge.

Next consider point K' inside the geometrical shadow region. At this point the displacement is due to the two unobstructed halves of wave front.

The half-period elements in the upper half will combine into a resultant whose phase will be in arrangement with the resultant phase of the half-period elements, lying near the edge A. Similarly the half period elements or zones in lower half will combine into resultant phase. These will be the same as the resultant phase of the half period elements lying near B.

The phase difference between the two resultants to Z is $\frac{dx}{D}$ where D is the distance between the pin and the screen and $x = cz$. Thus the point Z will be bright, if

$$\frac{dx}{D} = 2n\frac{\lambda}{2} \quad ...(1)$$

and dark, if

$$\frac{dx}{D} = (2n+1)\frac{\lambda}{2} \quad ...(2)$$

where n is the integer.

These maxima and minima obtained inside the geometrical shadow region resemble the fringes observed in an interference experiment. They are not due to diffraction. The fact they may be regarded as due to interference between the two narrow sources estimated at the edges of the obstacle. The distance between two consecutive dark and bright fringes, are called the fringe width W. It can be obtained with the help of eqn. (1) and eqn. (2).

Thus for $n = 1$ and $n = 2$, eqn. (1) gives bright fringes, as:

$$\frac{dx_1}{D} = \lambda \quad ...(3)$$

$$\frac{dx_2}{D} = 2\lambda \quad ...(4)$$

$$W = x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

or,

$$W = \frac{D\lambda}{d}$$

This eqn. can be used for determination of wavelength of light.

Manipulations:

1. Level the optical bench with the help of spirit level and leveling screw.
2. Make the slit vertical with the help of plumb line.
3. Bring the slit, the eyepiece at the same height and in the same line. See that the planes of the slit, the wire and eyepiece are transversely normal to the bench.
4. Focus the light by a lens on the slit.
5. Move the pin wide way with the help of tangential screws and vary the slit width till fringes are obtained and are seen through the eyepiece. The visibility will be best when the slit and the pin are parallel to each other.
6. Adjust the line joining the pin and the cross wires parallel to the bed of optical bench. This is accomplished when on moving the eyepiece along the bed of the optical bench, no lateral shift is obtained.
7. In order to adjust the system for no lateral shift, the eyepiece is moved away from the straight edge (pin). In this case the fringes will move to the right or left, but with the help of the base screw provided with pin (wire) it is moved at right angle to the bench in a direction to bring the fringes back to their original position,

   Now move the eyepiece towards the wire and same adjustment is made with the help of eyepiece.

   Using the process again and again the lateral shift is removed.
8. Measure the fringe width and the diameter of the pin.
9. Repeat the experiment with 2 more values of *D*.

Observations:

1. For diameter of pin.

   Least count of screw gauge =

   Zero error of screw gauge =

Readings	1	2	3	4	5	6	7	8	9	10
Along one direction (in cm.)										
⊥ to above (in cm.)										

   Mean diameter of a wire =

   Diameter corrected from zero error =

2. **Bench Error:** The distance between cross wire and pin:-

   (a) As measured by bench rod =

   (b) as measured by the bench scale =

   The bench error =

3. **Fringe Width:**

   Least count of Micrometer =

   Observed distance between the cross wire and the pin =

   Corrected distance between the cross wire and the pin =

No. of fringes	*Reading of Micrometer (in cm)*	*Width of 3 Fringes (in cm)*	*Mean (in cm)*	*Fringe width W (in cm)*
1 2 3 4 5 6				

Fringe width W =

Result: For sodium light (λ = 5893 Å) diameter '*d*' of a pin =
Actual diameter as measured by screw gauge =
Percentage error = ...%

Precautions:

1. The bench error is necessary therefore it should be found.
2. The straight pin should be parallel to the slit.
3. Make the slit as narrow as possible until the fringes are most clear.
4. The cross wire of the microscope should be well focused on the fringes.

14

Resolving Power

14.1 RESOLVING POWER

Due to diffraction, the image of a point object formed by an optical instrument has finite dimensions. It consists of a diffraction pattern, a central maximum surrounded by alternate dark and bright rings. Two point objects, are resolvable by an optical instrument if their diffraction pattern are sufficiently small or are far enough apart so that they can be distinguished as separate image patterns. The resolving power of an optical instrument is defined as its ability to produce separate and distinguishable images of two objects lying very close together. The diffraction effects set a theoretical limit to the resolving power of any optical instrument. The term resolving is used in two contexts.

14.2 GEOMETRICAL RESOLVING POWER

When the purpose is to see as separate two objects close together or when fine structure is seen through a telescope or microscope. In the case of telescope (or eye), the resolving power is defined as the smallest angle subtended at the objective of the telescope (or the eye) by two point objects which can be seen just separate and distinguishable. Smaller is this angle, the greater will be the resolving power of the instrument. For a microscope the resolving power is defined as the linear separation which the two neighbouring point objects can have and yet be observed as just separate and distinguishable when seen through the microscope.

14.3 CHROMATIC RESOLVING POWER

This term is used when the instrument such as prism or grating spectrometers are employed for spectroscopic studies. The purpose of these instruments is to disperse light emitted by a source and to produce its spectrum. The chromatic resolving power of an instrument is its ability to separate and distinguish between two spectral lines whose wavelengths are very close. Smaller the wavelength interval at a particular wavelength that can be separated, the greater is the resolving power. If a source emits two close wavelengths λ and $(\lambda + d\lambda)$, the resolving power is mathematically defined as the ratio $\frac{\lambda}{d\lambda}$ provided the wavelength interval $d\lambda$ can just be separated at the wavelength λ.

14.4 CRITERION FOR RESOLUTION ACCORDING TO LORD RAYLEIGH

Lord Rayleigh has set a criterion to decide as to how close the two diffraction patterns can be brought together such that the two images can just be recognised as separate and distinguished from each other. The criterion is applicable to both the geometrical as well as spectroscopic resolving powers. According to Rayleigh's criterion the two point sources are just resolvable by an optical instrument when their distance apart is such that the central maximum of the diffraction pattern of one source coincides in position with the first diffraction maximum of the diffraction pattern of the other source. When applied to the resolution of spectral lines, this principle is equivalent to the condition that for just resolution the angular separation between the principle maxima of the two spectral lines in a given order should be equal to half angular width of either of the principal maximum. In this latter case, it is assumed that the two spectral lines have equal intensities.

In Fig. 14.1(a), *A* and *B* are the central maxima of the diffraction patterns of two spectral lines of wavelengths λ_1 and λ_2. The difference in the angle of diffraction is large and the two images can be seen as separate ones. The angle of diffraction corresponding to the central maximum of the image *B* is greater than the angle of diffraction corresponding to the first minimum at the right of *A*. Hence the two spectral lines will appear well resolved. In Fig. 14.1(b) the central maxima corresponding to the wavelength λ and $\lambda + d\lambda$ are very close. The angle of diffraction corresponding to the first minimum of *A* is greater than the angle of diffraction corresponding to the central maximum of *B*. Thus, the two images overlap and they cannot be distinguished as separate images. The resultant intensity curve gives a maximum as at *C* and the intensity of this maximum is higher than the individual intensities of *A* and *B*. Thus when the spectrograph is turned from *A* to *B*, the intensity increases, becomes maximum at C and then decreases. In this case, the two spectral lines are not resolved.

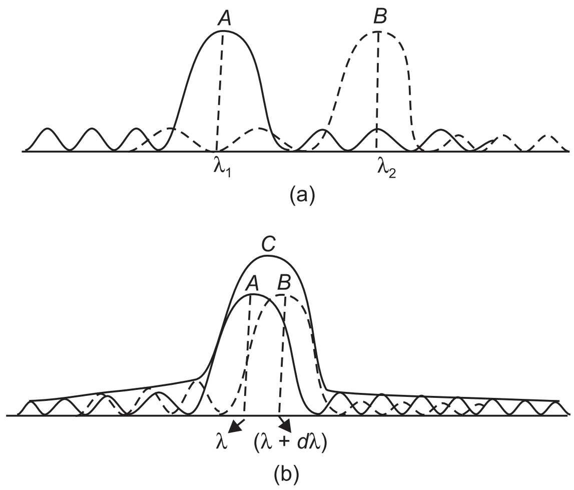


Fig. 14.1

In Fig. 14.1(c), the position of the central maximum of *A* (wavelength λ) coincides with the position of the first minimum of *B*. (wavelength $\lambda + d\lambda$). Similarly, the position of the central maximum of *B* coincides with the position of the first minimum of *A*. Further, the resultant intensity intensity curve shows a dip at *C* i.e., in the middle of the central maxima of *A* and *B* (Here, it is assumed that the two spectral lines are of the same intensity). The intensity at *C* is approximately 20% less than that at *A* or *B*. If a spectrograph is turned from the position corresponding to the central image of *A* to the one corresponding to the image of *B*, there is noticeable decrease in intensity between the two central maxima. The spectral lines can be distinguished from one another and according to Rayleigh they are said to be just resolved Rayleigh's condition can also be stated as follows. Two images are said to be just resolved if the radius of the central disc of either pattern is equal to the distance between the centers of the two patterns.

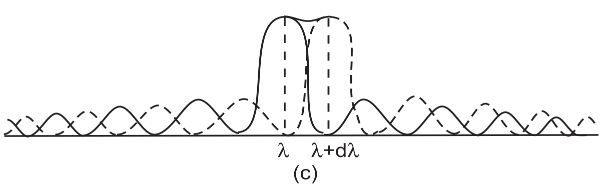


Fig. 14.1

14.5 RESOLVING POWER OF A TELESCOPE

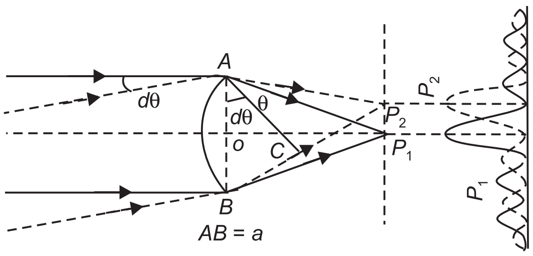


Fig. 14.2

Let a be the diameter of the objective of the telescope. Consider the incident ray of light from two neighbouring points of a distant object. The image of each point object is a Fraunhofer diffraction pattern. Let P_1 and P_2 be the positions of the central maxima of the two images. According to Rayleigh, these two images are said to be resolved if the position of the central maximum of the second image coincides with the first minimum of the first image and vice versa. The path difference between the secondary waves travelling in the directions AP_1 and BP_1 is zero and hence they reinforce one another at P_1. Similarly, all the secondary waves from the corresponding points between A and B will have zero path difference. Thus, P_1 corresponds to the position of the central maximum of the first image.

The secondary waves travelling in the directions AP_2 and BP_2 will meet at P_2 on the screen. Let the angle P_2AP_1 be $d\theta$. The path difference equal to BC.

From the $\Delta\, ABC$

$$BC = AB \sin d\theta$$

for small angles

$$BC = AB \cdot d\theta = a \cdot d\theta$$

If this path difference $a \cdot d\theta = \lambda$, the position of P_2 corresponds to the first minimum of the first image. But P_2 is also the position of the central maximum of the second image. Thus, Rayleigh's condition of resolution is satisfied if

$$a \cdot d\theta = \lambda$$

or

$$d\theta = \frac{\lambda}{a} \qquad ...(i)$$

The whole aperture AB can be considered to be made up of two halves AO and OB. The path difference between the secondary waves from the corresponding points in the two halves will be $\frac{\lambda}{2}$. All the secondary waves destructively interfere with one another and hence P_2 will be the first minimum of the first image. The equation $d\theta = \frac{\lambda}{a}$ holds good for rectangular aperture. For circular aperture, this equation, according to Airy, can be written as

$$d\theta = \frac{1.22\,\lambda}{a} \qquad ...(ii)$$

where λ is the wavelength of light and a is the aperture of the telescope objective. The aperture is equal to the diameter of the metal ring in which the objective lens is mounted. Here $d\theta$ refers to the limit of resolution of the telescope. The reciprocal of $d\theta$ measures the resolving power of the telescope.

$$\therefore \qquad \frac{1}{d\theta} = \frac{a}{1.22\,\lambda} \qquad ...(iii)$$

From Eq. (iii), it is clear that a telescope with large diameter of the objective has higher resolving power, $d\theta$ is equal to the angle subtended by the two distant object points at the objective.

Thus resolving power of a telescope can be defined as the reciprocal of the angular separation that two distant object points must have, so that their images will appear just resolved according to Rayleigh's criterion.

If f is the focal length of the telescope objective, then

$$d\theta = \frac{r}{f} = \frac{1.22\,\lambda}{a}$$

or

$$r = \frac{1.22\,f\lambda}{a} \qquad ...(iv)$$

where r is the radius of the central bright image.

The diameter of the first dark ring is equal to the diameter of the central image. The central bright disc is called the Airy's disc.

From equation (iv), if the focal length of the objective is small, the wavelength is small and the aperture is large, then the radius of the central bright disc is small. The diffraction patterns will appear sharper and the angular separation between two just resolvable point objects will be smaller. Correspondingly, the resolving power of the telescope will be higher.

Let two distant stars subtend an angle of one second of an arc at the objective of the telescope.

1 second of an arc = 4.85×10^{-6} radian. Let the wavelength of light be 5500 Å. Then, the diameter of the objective required for just resolution can be calculated from the equation

$$d\theta = \frac{1.22\lambda}{a}$$

or

$$a = \frac{1.22\lambda}{d\theta} = \frac{1.22 \times 5500 \times 10^{-8}}{4.85 \times 10^{-8}} = 13.9 \text{ cm. (approximately)}$$

The resolving power of a telescope increases with increase in the diameter of the objective. With the increase in the diameter of the objective, the effect of spherical aberration becames appreciable. So, in the case of large telescope objectives, the central portion of the objective is covered with a stop so as to minimize the effect of spherical aberration. This, however, does not affect the resolving power of the telescope.

14.6 OBJECT

To determine the resolving power of telescope.

Apparatus Used: A telescope fitted with a variable width rectangular aperture to its objective, a sodium lamp, three pair of slits a focusing lens, two mountings for the slit pair and the lens.

Formula Used: The theoretical and practical resolving power are given by theoretical resolving power = λ/D and practical resolving power = u/x

Where λ = mean wavelength of light employed.

D = width of the rectangular slit for just resolution of two objects

x = separation between two objects.
u = distance of the objects from the objective of the telescope.

Theory: The resolving power of a telescope is defined as the inverse of the least angle subtended at the objective by two distant point objectives (of equal brightness) which can just be distinguished as separate in its focal plane.

Let the parallel ray of light from two distant objects subtend an angle (θ) at the telescope objective *AOB*. The image of each point object is a fraunhoffer diffraction pattern consisting of a central bright disc surrounded by concentric dark and bright rings due to circular aperture of the objective. The diffraction patterns overlap each other and the two images will just be resolved. According to Rayleigh's criterion, on the first minimum of one image coincide with the central maximum of the other and vice versa.

According to theory the least resolvable angle θ is given by λ/D and the resolving power of the telescope.

$$1/\theta = D/\lambda$$

The resolving power (experimentally) is also given by

$$1/\theta = 1/x/u = u/x$$

Where 'x' is the distance between the two line objects and 'u' is the distance from telescope objective, if θ is the angle when the two images are just resolved.

Method:

1. The focusing lens and a pair of slits are mounted on their respective stands. The slits are made vertical with the help of a plumb line by using the screw attached to the stand.
2. Light from the sodium lamp is focused on the slits by means of the lens.
3. The axis of the telescope is made horizontal by means of spirit level and its height is so adjusted that the images of the pair of slits are symmetrical with respect to the cross point of the cross wires. The inter adjustment can also be obtained by keeping the variable aperture wide open and adjusting the telescope.
4. The images are brought into sharp focus by adjusting the telescope while keeping the variable aperture wide open.
5. The width of the aperture is gradually reduced so that at first the two images appear out and ultimately their separation vanishes. The width of the aperture at this critical position may be measured by means of a micrometer screw or the readings are noted directly on the vernier scale attached to the aperture. Reducing aperture further we note the reading when the illumination (light) just disappears altogether. The difference of these two readings gives width of the aperture required.
6. Now we begin with a closed aperture gradually increasing the width. We take the first reading when the illumination just appears and then when the two images just appear to be separated. The difference giving the width of the aperture is noted.
7. The operation 5 and 6 are repeated.
8. The operation 1 to 7 is repeated for the other two pairs of slits.
9. The distance between the slits (sources) and the objective of the telescope is measured by means of measuring tape.

Observation:

1. Least count of the micrometer attached to the variable width aperture =
2. Minimum width of aperture for resolution D =

3. Distance between the objective of the telescope and the slit sources (u) =
4. Wave length of the light employed =

Pair of Slits	Distance between the slits (in cm.) x	Micrometer reading while aperture width decreasing (cm.)	Micrometer reading while aperture width increasing (cm.)	Mean (in cm.) D
A.				
B.				
C.				

Calculations:

Pair of slits	Theoretical resolving power $T = D/\lambda$	Experimental resolving Power $E = u/x$	% Difference
A.			
B.			
C.			

Result: The resolving power of telescope as measured is nearly equal to the theoretical value.

Precautions:

1. The axis of telescope should be horizontal.
2. The screw attached to the variable width aperture should be handled gently. While decreasing the width of the aperture we should stop at the point when illumination just disappears. We should not tight the screw beyond this point. This should be the starting point taking for readings for the increasing aperture.
3. Backlash error in the micrometer screw should be avoided.
4. The axis of the telescope should be at right angles to the plane containing the slits. The two slits should appear equal in height.
5. Care should be taken that distance between the lens and the slit source is more than focal length of lens (about 30 cm).

14.7 VIVA-VOCE

Q. 1. What do you mean by resolving power of a telescope?

Ans. The resolving power of a telescope is defined as the reciprocal of the smallest angle subtended at the objective by two distinct points which can be just seen as separate through the telescope.

Q. 2. On what factors does the resolving power of a telescope depend?

Ans. The resolving power of a telescope is given by

$$\frac{1}{d\theta} = \frac{d}{1.22\,\lambda}$$

Resolving power is directly proportional to d i.e., a telescope with large diameter of objective has higher resolving power and inversely proportional to λ.

Q. 3. Why are the telescopes fitted with objectives of large diameter ?

Ans. To increase the resolving power of telescope.

Q. 4. Does the resolving power of a telescope depend upon the focal length of its objective?

Ans. No.

Q. 5. Does any thing depend on *f*?

Ans. Yes, Magnifying power increases with *f*.

Q. 6. Sometimes an observer gets a higher than the theoretically expected value of resolving power. How do you explain it?

Ans. It is because that Rayleigh criterion is itself quite arbitrary and skilful experimenters can exceed the Rayleigh limit.

Q. 7. Define the magnifying power of the telescope.

Ans. The magnifying power of a telescope is defined as the ratio of angle subtended at the eye by the final image and the angle subtended at the eye by object when viewed at its actual distance.

Q. 8. What is Rayleigh criterion of resolution?

Ans. According to Rayleigh criterion, two point sources are resolvable by an optical instrument when the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction pattern of the other and vice-versa.

Q. 9. What does the term 200 inch written on a telescope indicate?

Ans. This indicate that the diameter of the objective of the telescope is 200 inches.

EXERCISE

Q. 1. What do you mean by resolving limit of a telescope?

Q. 2. What is the resolving power of the eye?

Q. 3. How does the minimum angle of resolution change by putting variable aperture before the telescope objective?

Q. 4. Does the resolving power of a telescope depend upon the distance between the telescope and the objects to be resolved?

Q. 5. Is it possible to attain the theoretical resolving power?

Q. 6. What will be the resolving power of this telescope?

9

Thermoelectric Effect

9.1 THERMOELECTRIC EFFECT

Seebeck discovered the thermoelectric effect. To study this effect, two wires of different materials say copper and iron are joined at their ends so as to form two junctions. A sensitive galvanometer is included in the circuit as shown in Figure 9.1. This arrangement is called a Cu-Fe thermocouple. When one junction of the thermocouple is kept hot and the other cold, the galvanometer gives deflection indicating the production of current in the arrangement. The current so produced is called thermoelectric currents.

The continuous flow of current in the thermocouple indicates that there must be a source of e.m.f. in the circuit, which is causing the flow of current. This e.m.f is called thermoelectric e.m.f. It is found that for a temperature difference of 100ºC between the hot and cold junction, thermo e.m.f. produced in Cu-Fe thermocouple is 0.0013V and in case of Sb-Bi thermocouple, the thermo e.m.f. produced is 0.008V.

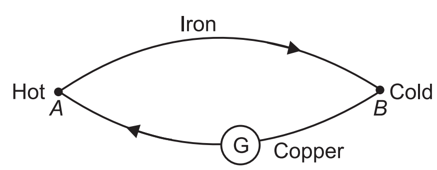


Fig. 9.1

This phenomenon of production of electricity with the help of heat is called thermoelectricity and this effect is called thermoelectric effect or Seebeck effect.

Thus, the phenomenon of production of e.m.f. causing an electric current to flow in a thermocouple when its two junctions are kept at different temperature, is known as Seebeck effect.

9.2 ORIGIN OF THERMO E.M.F.

In a conductor, there are always free electrons. In any conductor, the number of free electrons per unit volume (*electron* density) depends upon its nature. In general, the electron density increases with rise in temperature.

When two metallic wires of different materials are joined at their ends to form a thermocouple, electrons from a metal having greater electron density diffuse into the other with lower value of electron density. Due to this, a small potential difference is established across the junction of the two metals. The potential difference so established is called contact potential and its value depends upon the temperature of the junction for the two given metal obviously, if the two junctions are at the same temperature, the contact potentials at the two junctions will be equal. As the contact potentials at the two junctions tend to send current in opposite directions, no current flows through the thermocouple.

However, if one of the two junctions is heated, more diffusion of electrons takes place at the hot junction and the contact potential becomes more than that at the cold junction. Hence, when the two junctions of a thermocouple are at different temperature, a net e.m.f. called thermo e.m.f. is produced.

It may be pointed out that Seebeck effect is reversible. If we connect a cell in the circuit so as that it sends current in a direction opposite to that due to Seebeck effect Figure 9.2, then it is observed that heat is rejected at the hot junction and absorbed at the cold junction i.e. the hot junction will start becoming hotter, while the cold junction still colder.

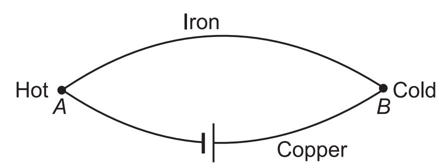


Fig. 9.2

A thermo couple also acts like a heat engine. It constantly absorbs heat at the hot junction, rejects a part at the cold junction and the remaining part is converted into electrical energy, which sends current through that thermocouple.

9.3 MAGNITUDE AND DIRECTION OF THERMO E.M.F.

The magnitude and direction of the thermo e.m.f. developed in a thermocouple depends upon the following two factors.

(*i*) **Nature of the metals forming the thermocouple:** For the experimental investigations, Seebeck arranged a number of metals in the form of a series called thermoelectric series. Some of the metals forming this series are as below:

Sb, Fe, Zn, Ag, Au, Mo, Cr, Sn, Pb, Hg, Mn, Cu, Pt, Co, Ni and Bi.

If a thermocouple is formed with wires of any two metals from this series, the direction of current will be from a metal occurring earlier in this series to a metal occurring later in the series through the cold junction. Therefore, in copper-iron (Cu-Fe) thermocouple, the current will flow from iron to copper through cold junction or Copper to Iron through the hot junction. In antimony-bismuth (Sb-Bi) thermocouple, the current flows from antimony to bismuth through the cold junction.

The thermo e.m.f. for a difference of temperature equal to 100ºC is about 0.0013V for Cu-Fe thermocouple and about 0.008V for Sb-Bi thermocouple. As a rule, more the metals are separated in the series, the greater will be the thermo e.m.f.

(*ii*) **The temperature difference between the two junctions of the thermocouple:** To study the effect of difference of temperature between the two junctions, consider a Cu-Fe thermocouple. Its one junction is kept hot by immersing in oil bath and heated with burner.

The other junction is kept cold by immersing it in pounded ice. The temperature of the hot junction can be measured by the thermometer *T* placed in the hot oil bath.

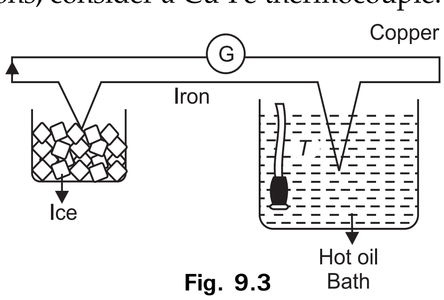


Fig. 9.3

As the temperature of the hot junction is increased by keeping the temperature of the cold junction constant at 0ºC, the deflection in the galvanometer goes on increasing. The deflection in the galvanometer is directly proportional to the thermoelectric current and hence the thermo e.m.f. The graph between thermo e.m.f. and the temperature of hot junction is found to be parabolic in shape as shown in Figure.

As the temperature of hot junction is further increased, a stage comes, when the thermo e.m.f. becomes maximum.

The temperature of hot junction at which the thermo e.m.f. produced in the thermocouple becomes maximum, is called neutral temperature. For a given thermocouple, neutral temperature has a fixed value. It does not depend upon the temperature of cold junction of the thermocouple. It is denoted by θ_n. For copper-iron thermocouple, neutral temperature is 270ºC.

The temperature of the hot junction, at which the direction of the thermo e.m.f. reverses, is called the temperature of inversion. It is denoted by θ_i.

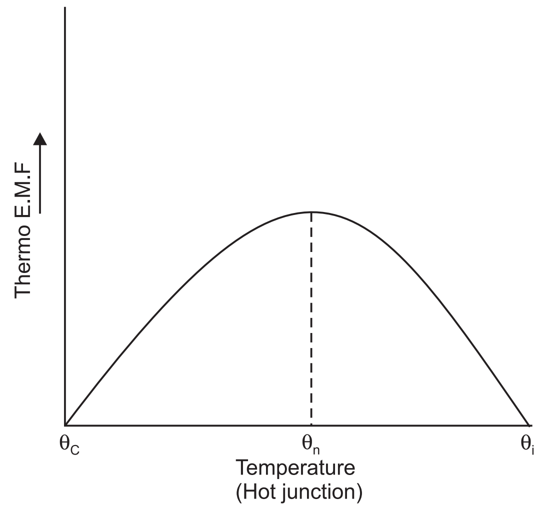


Fig. 9.4

The temperature of inversion is as much above the neutral temperature as the neutral temperature is above the temperature of the cold junction. Then, if θ_c is temperature of the cold junction, then

$$\theta_n - \theta_c = \theta_i - \theta_n$$

or

$$\theta_n = \frac{\theta_i + \theta_c}{2} \quad \text{and} \quad \theta_i = 2\,\theta_n - \theta_c$$

Thus, the netural temperature is the mean of the temperature of inversion θ_i and temperature of the cold junction θ_c, but is independent of θ_i and θ_c. For Cu-Fe thermocouple θ_n = 270ºC. If cold junction is at 0°C, then it follows that θ_i = 540ºC.

If the temperature of cold junction is 0ºC, the graph between the temperature of hot junction and thermo e.m.f. is found to satisfy the equation of the parabola.

$$E = \alpha\theta + \beta\theta^2$$

Where α and β are constants called thermoelectric constants. θ represents the temperature difference between the hot and cold junctions.

9.4 PELTIER EFFECT

Let us consider a bismuth-copper (Bi-Cu) thermocouple. Due to Seebeck effect, in such a thermocouple, thermo electric current flows from copper to bismuth through cold junction. Heat is absorbed at hot junction and is evolved at cold junction.

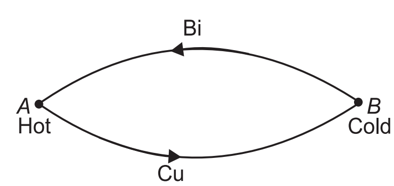


Fig. 9.5

Peltier discovered that whenever two dissimilar metals are connected at a point, an electromotive force (e.m.f.) exists across the junction. Thus, out of the two metals, one is at higher potential than the other. This e.m.f. is found to vary with the change in temperature of the junction.

In the Bi-Cu thermocouple, copper is at higher potential as compared to bismuth. Thus, the Bi-Cu thermocouple appears as if two cells are connected across the two junctions, with their positive terminals to the ends of copper wires. Further, this contact e.m.f. of the cell across the hot junction is greater than that across the cold junction. The net e.m.f. causes the current to flow from copper to bismuth through the hot junction. At the hot junction, as the current flows from Bi to Cu i.e. from lower potential to higher potential, energy will be needed for this purpose. For this reason, in a thermocouple, heat is absorbed at the hot end. On the other hand, energy is given out in the form of heat at the cold junction, as here the current flows from higher to lower potential.

This absorption or evolution of heat at a junction of two dissimilar metals, when current is passed, is known as Peltier effect.

It is also a reversible phenomenon. If the direction of flow of current is reversed, then at a junction heat will be evolved, if earlier heat was absorbed there and vice-versa.

9.5 PELTIER COEFFICIENT(π)

The amount of heat energy absorbed or evolved per second at a junction, when a unit current is passed through it, is known as Peltier Coefficient. It is denoted by π.

Suppose cold junction is at temperature T and hot junction at $T + dT$. If dE is the thermo e.m.f. produced, then it is found that

$$\frac{\pi}{T} = \frac{dE}{dT}$$

Here, $\frac{dE}{dT}$ is rate of change of thermo e.m.f. with temperature. It is called thermoelectric power. It is also known as Seebeck coefficient. If S is Seebeck Coefficient, then

$$S = \frac{dE}{dT}$$

Thus

$$\boxed{\frac{\pi}{T} = \frac{dE}{dT} = S}$$

9.6 THOMSON'S EFFECT

Thomson found that in a Copper wire whose one end is hot and the other kept cold, if current is passed from hotter end to colder end, then heat is evolved along the length of the copper wire. In case, current is passed from colder end to the hotter end, then heat is absorbed along the length of the Copper wire, The explanation lies in the fact that in case of Copper wire, the hot end is at higher potential and the cold end is at lower potential. When current flows from hotter to colder end i.e. from higher to lower potential, the energy is given out in the form of heat. On the other hand, when current is passed from colder to hotter end i.e. from lower to higher potential, the energy is required and it leads to absorption of the heat energy.

In case of bismuth, the effect is just reverse i.e. heat is evolved along the length of bismuth wire, when current is passed from colder to hotter end and heat is absorbed, when current is passed from hotter to colder end. It is because, in case of bismuth, the hot end is at lower potential and cold end is at higher potential.

This absorption or evolution of heat along the length of a wire, when current is passed through a wire whose one end is hot and other is kept cold, is known as Thomson effect. Thomson effect is also a reversible phenomenon. The substances which behave like Copper are said to have positive Thomson effect. Such other substances are antimony, silver, zinc, etc. On the other hand, substances such as cobalt, iron, platinum, etc. which behave like bismuth are said to have negative Thomson effect.

9.7 THOMSON COEFFICIENT

The amount of heat energy absorbed or evolved per second between two points of a conductor having a unit temperature difference, when a unit current is passed, is known as Thomson Coefficient for the material of a conductor. It is denoted by σ.

Thomson coefficient of the material of a conductor is found by forming its thermocouples with a lead wire (Thomson Coefficient of lead is zero). It can be proved that Thomson Coefficient of the material of conductor is given by

$$\sigma = -T\frac{d^2E}{dT^2}$$

Now, Seebeck coefficient is given by

$$S = \frac{dE}{dT}$$

$$\therefore \qquad \frac{dS}{dt} = \frac{d^2E}{dT^2}$$

Thus
$$\sigma = -T\frac{d^2E}{dT^2} = -T\left(\frac{dS}{dT}\right)$$

9.8 THERMOPILE

Principle: For a given small difference in temperature of two junctions of a thermocouple, Bi-Sb thermocouple produces a comparatively large e.m.f. and it can be used to detect the heat radiation. When, a number of such thermocouples are connected in series, the arrangement becomes very much sensitive to detect heat radiation as the thermo e.m.f.'s of the thermocouples get added.

A series combination of a large number of Bi-Sb thermocouples is enclosed in a funnel or horn-shaped vessel.

The junction *A* of each thermocouple is coated with lamp black while the junction *B* of each thermocouple is well polished and covered with insulating material. The extreme ends of the arrangement are connected to the terminals T_1 and T_2, across which a sensitive galvanometer is connected.

Such an arrangement known as thermopile is shown in Fig. 9.6.

When heat radiations fall on the funnel shaped end of the thermopiles, the set of junction *B* coated with lamp black absorbs the heat radiation. As a result, the temperature of set of junction *B* relative to junctions *A* get raised and thermo e.m.f. is developed in each thermocouple.

The thermoelectric current flows in the same direction (from Sb to Bi through cold junction) in all the thermocouples. Therefore, a large current flowing through the circuit produces deflection in the galvanometer, which indicates the existence of heat radiation.

The thermoelectric effect has the following important applications:

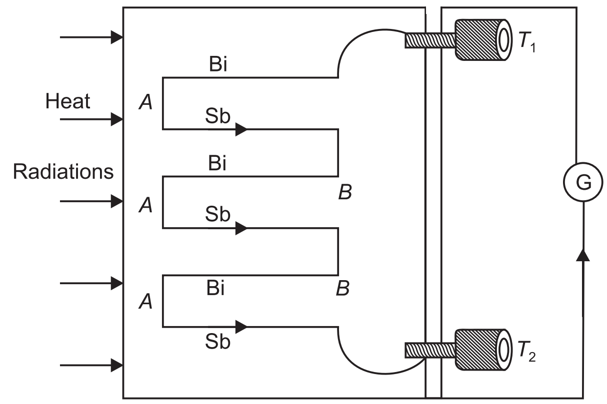


Fig. 9.6

1. A thermocouple is preferred and used to measure temperatures in industries and laboratories. One junction is kept cold at known temperature and other junction is placed in contact with the object, whose temperature is to be measured. The temperature is calculated from the measured value of the thermo e.m.f. The thermocouple is preferred to measure temperature for the following reasons:

(*a*) Since the junction is very small, it absorbs only a very small heat and therefore it does not change the temperature of the object.
(*b*) It quickly attains the temperature of the object.
(*c*) The accuracy in the measurement of temperature is very high. It is because the measurements are made of electrical quantities.

The type of the thermocouple to be used is determined by the range of measurement of the temperature. Different types of thermocouples used in different ranges of temperature are given below:

Thermocouple	Temperature range
Copper-gold and Iron alloy	1 K to 50 K
Copper-constantan	50 K to 400 K
Platinum-platinum rhodium alloy	1500 K to 2000 K.

2. **To detect heat radiation:** A thermopile is a combination of large number of thermocouples in series. It can be used to detect the heat radiation and to note the small difference or the variation in temperatures.

3. **Thermoelectric refrigerator:** If a current is passed through a thermocouple, then due to Peltier effect, heat is removed at one junction and is absorbed at other junction. In case, if on the whole, heat is removed, then the thermocouple acts as a thermoelectric refrigerator. The advantage is that it has, no compressor. No doubt, the cooling effect produced is much low as compared to that in the case of conventional refrigerators. A thermoelectric refrigerator is used, when the region to be cooled is very small and the noise is not acceptable. The thermocouple used as a thermoelectric refrigerator should have following three characteristics:

(*a*) It should have low resistivity, otherwise, loss of energy in the form of heat will be large.
(*b*) It should have low thermal conductivity. It will help in maintaining large temperature difference between the two junctions.
(*c*) It should produce high thermopower.

4. **Thermoelectric generator:** Thermocouples can be used to generate thermoelectrical power in remote areas. It may be done by heating one junction in a flame and exposing the other junction to air. The thermo e.m.f. developed has been used to power radio receivers, etc.

9.9 OBJECT

To calibrate a thermocouple and to find out the melting point of naphthalene.

Apparatus used: One thermocouple apparatus as shown in the Fig. 9.7, a galvanometer, two thermometers, key, Naphthalene, stop watch and heater or gas burner, glass test tube, stand and clamp.

Description of apparatus and theory: The apparatus as shown in the diagram consists of two junctions *A* and *B* of copper and constantan. These are placed in the test tubes which themselves are placed in the baths B_1 and B_2. One of them is kept at room temperature are measured by thermometers T_1 and T_2.

When a difference of temperature is produced between two junctions an E.M.F. is set up which produces a deflection in the galvanometer. This deflection is proportional to the difference of temperature between the two junctions. Thus by plotting a graph with known difference of temperature and known deflections we can find out an unknown temperature by noting its deflection and finding out the corresponding temperature from the graph.

Manipulations:

1. Set the apparatus as shown in the diagram.
2. Put junction *A* in cold water and note temperature T_1.
3. Put junction *B* in cold water and heat the water.
4. Start observations from the room temperature and take galvanometer readings at intervals of 4ºC – 5ºC and go up to the boiling point of water. To keep the temperature constant for some time, remove the flame at the time of taking observation and keep on a stirring the water.
5. Plot a graph with deflection as ordinate and temperature as abscissa. The graph, in general, should be a parabola (Fig. 9.8(*a*)) but within a short range of temperature as in the present case (0ºC – 90ºC or 100ºC), the graph will be straight line Fig. 9.8(*b*). The straight line actually is the straight portion of the parabola. This graph can be used to determine any unknown temperature (within this range).
6. Pour the naphthalene bits into the test tube and immerse the end *B* of the thermocouple in it keeping the test tube immersed in hot water in a water bath.

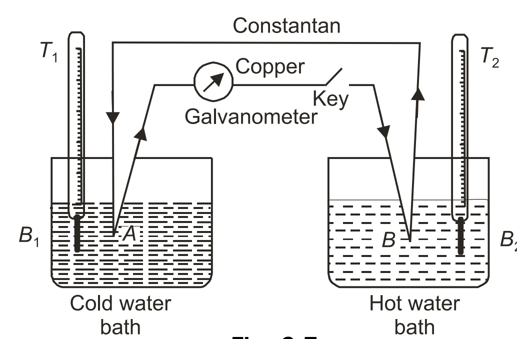


Fig. 9.7

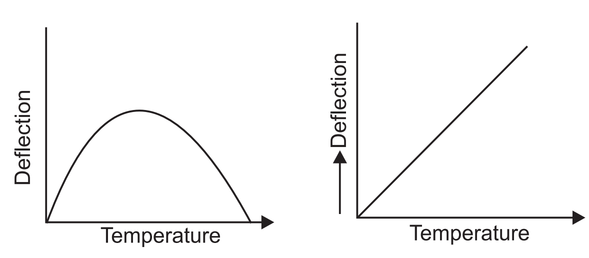


Fig. 9.8

7. Find out the deflection when the naphthalene melts and also when it solidifies again. The deflection should be read at the intervals of 30 seconds. When the naphthalene is about to melt or solidify, the deflection will remain constant during melting and solidification. Note the value of constant deflection.
8. Find out from the previous graph, the value of temperature corresponding to the deflection. This is the melting point of naphthalene.

Observation:

Temperature of the cold junction = _ _ _ _ºC

Initial reading of galvanometer =

(1) With water in B

Sl No.	*Temperature of the hot junction*	*Reading for deflections*		*Mean Deflection*	*Net Deflection*
		While heating	*While cooling*		
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
9.					
10.					

(2) With Naphthalene in B

Sl No.	*Reading for deflection while melting*	*Reading for deflection while solidify*	*Mean of constant readings*	*Net deflection*
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				

9.				
10.				
11.				
12.				
13.				
14.				
15.				
16.				

The temperature of melting naphthalene corresponding to this deflection from the graph =

Result: The melting point of the Naphthalene =

Standard value: The melting point of Naphthalene =
Percentage error =

Precautions:

1. The reading of galvanometer should be taken carefully.
2. The hot junction should be carefully dipped in naphthalene.
3. Do not inhale naphthalene vapor, as it may be harmful.
4. The ends of connecting wires should be properly cleaned.
5. The wires forming thermocouple should be in contact with each other at the junction only.

9.10 VIVA-VOCE

Q. 1. On what factors does the direction of thermo electric current depends?
Ans. It depends upon the nature of metals in contact.

Q. 2. What is the direction of current in case of copper-iron and antimony-bismuth couple?
Ans. For copper-iron couple-current flows from copper to iron at hot junction. For antimony-bismuth couple the current flow at cold junction from antimony to bismuth.

Q. 3. What is thermo-couple?
Ans. A thermo couple is a circuit formed by joining two dissimilar metals. Its junctions are kept at different temperature.

Q. 4. What is neutral temperature?
Ans. The neutral temperature is the temperature at which thermoelectric e.m.f. is maximum.

Q. 5. How does the temperature of inversion vary?
Ans. The temperature of inversion for a given couple at hot junction is as much above the neutral temperature as the temperature of cold junction is below it.

Q. 6. What is thermoelectric effect?
Ans. When two junctions of different metals are kept at different temperatures, then an emf produced in the circuit gives rise to a current in the circuit. It is called Seebeck effect.

Q. 7. Is the neutral temperature same for all thermocouples?
Ans. No, it is different for different thermocouples.
Q. 8. What is temperature of inversion?
Ans. The temperature at which thermoelectric emf changes its sign, is called temperature of inversion.
Q. 9. How temperature of inversion θ_i is related to neutral temperature θ_n?
Ans. $\theta_n - \theta_c = \theta_i - \theta_n$
Q. 10. What is calibration curve?
Ans. A curve showing the variation of thermoelectric emf with temperature, is known as calibration curve.
Q. 11. What is Peltier effect?
Ans. When current is passed through the junction of two different metals, one junction is heated while other is cooled.
Q. 12. How thermo emf is generated?
Ans. The concentration of electrons at the interface of two metals is different. The electrons from higher concentration interface, are transferred to lower concentration interface. Thus a constant potential difference is developed when one junction is hot and other is cold. The contact potential is higher at hot junction than that of cold junction, and so thermo emf is generated.
Q. 13. What is Thomson effect?
Ans. Whenever the different parts of the same metal are at different temperature an emf is developed in it. This is called Thomson effect.
Q. 14. What is Seebeck effect?
Ans. When two wires of different metals are joined at their ends and a temperature difference is maintained between the junctions, a current will flow in the circuit. This is known as Seebeck effect.
Q. 15. What is Peltier effect?
Ans. When a battery is inserted in a thermocouple circuit whose two junctions are initially at the same temperature, one of the junctions will became hot and the other cold. This phenomenon is known as Peltier effect.
Q. 16. Explain the existence of an emf at the junction of two metals.
Ans. When two different metals are joined at their ends the free electrons of one metal will flow to the other because the electron density of the two metals is different. As a result of the electron flow, one metal becomes positive with respect to the other and a potential difference is created at the junction. The magnitude of this potential difference, referred to as the contact potential difference, increases with temperature.
Q. 17. Explain the difference between Joule effect and Peltier effect.
Ans. In the case of Joule effect, the generated heat is proportional to the square of the current and is, therefore, independent of the direction of the current. Peltier effect, on the other hand, produces heating or cooling at a junction that is proportional to the current. Thus, a junction which is heated by a current will be cooled when the direction of the current is reversed.
Q. 18. How is a thermocouple constructed? Name some pairs of metals that are generally used for the construction of thermocouples.

Ans. To construct a thermocouple, say copper-constantan, one piece of constantan wire and two pieces of copper wires are taken. After cleaning the ends with emery paper, one end of each of the copper wires are spot-welded with the ends of the constantan wire forming two junctions of the thermocouple. Copper-constantan, copper-iron, platinum-rhodium etc. are the pairs of metals which are used for thermocouples.

Q. 19. Practical applications of thermocouples.

Ans. (*i*) To measure the temperature at a point.

(*ii*) To measure radiant heat.

Q. 20. Can you use an ordinary voltmeter to measure the thermo-emf?

Ans. No, because the emf is in the millivolt range.

Q. 21. What is the general nature of the thermo e.m.f. vs temperature curve? What is the nature of the curve that you have obtained?

Ans. Parabola. We obtain a straight line, because the temperature of the hot junction is much removed from the neutral temperature of the couple. That is, we obtain the straight portion of the parabola.

Q. 22. What is thermo-electric power? What is its value at 60°C for the copper-constantan thermocouple?

Ans. It is defined as the increase in thermo emf. of a thermocouple at a particular temperature of the hot junction per unit degree rise in temperature of the hot junction. For a copper-constantan thermocouple $\frac{dE}{dT}$ at 60ºC is about 40 μv per ºC.

Q. 23. Why does the null point remain constant during melting or freezing of the solid?

Ans. During melting or freezing the temperature does not change. This gives a constant null point reading.

Q. 24. What is the value of the thermo-electric power at the neutral temperature?

Ans. Zero.

Q. 25. How will the emf change if the temperature of the hot junction be increased beyond the neutral temperature?

Ans. The emf will decrease with increase of temperature and at a temperature, called the temperature of inversion, the e.m.f. will be reduced to zero. After the inversion temperature the polarity of thermo-emf will reverse.

Q. 26. What is the difference between heat and temperature?

Ans. The quantity of thermal energy present in a body is called 'heat' whereas the degree of hotness of a body is given by temperature.

Q. 27. Can you explain it on the kinetic theory?

Ans. The total kinetic energy possessed by all molecules of a body gives us the idea of 'heat' while the average kinetic energy possessed by a molecule gives us the 'temperature' of the body. It is given by $\frac{1}{2}m\overline{C}^2$ where m is the mass and $\overline{C}$ is the average velocity of the molecule.

Q. 28. How can you measure heat and temperature?

Ans. Heat is measured by the product of mass, specific heat and the temperature of the body. Temperature is measured by instruments called thermometers.

Q. 29. What types of thermometers do you know?

Ans. *Mecury thermometer:* These can be used for temperatures from –40ºC to 359ºC the freezing and boiling points of mercury.

Alcohol thermometers: These gives the maximum and minimum temperatures of the day and are used in meteorological departments.

Gas thermometers: They can be used from –260ºC to 1600ºC and are used to standardize the mercury thermometers.

Platinum resistance thermometers have the range from –200ºC to 1200ºC approximately.

Thermo-couples are also used to measure temperatures in the range –200ºC to 1700ºC.

Optical pyrometers are used to measure high temperatures from about 600ºC to 6000ºC.

Q. 30. Define specific heat?

Ans. It is defined as the ratio of the quantity of heat required to raise the temperature of a given mass of a substance to the quantity of heat required by the same amount of water to raise its temperature by the same amount.

Q. 31. What is a calorie?

Ans. It is the amount of heat in C.G.S. units required to raise the temperature of one gram of water from 14.5ºC to 15.5ºC.

Q. 32. Name the metals forming the thermoelectric series.

Ans. The following metals form the thermoelectric series:

Sb, Fe, Zn, Ag, Au, Mo, Cr, Sn, Pb, Hg, Mn, Cu, Co, Ni and Bi.

Q. 33. How does the thermoelectric series enable us to know the direction of flow of current in a thermocouple?

Ans. In the thermocouple formed of the two metals from the thermoelectric series, the current flows from the metal occurring earlier in the series to the metal occurring latter in the series through the cold junction.

Q. 34. Give the direction of thermo electric current: (*i*) at the cold junction of Cu-Bi (*ii*) at the hot junction of Fe-Cu (*iii*) at the cold junction of platinum-lead thermocouple.

Ans. (*i*) From Cu to Bi, (*ii*) from Cu to Fe, (*ii*) From lead to platinum.

Q. 35. How does the thermo e.m.f. vary with the temperature of the hot junction?

Ans. The thermo e.m.f. increases with increase in temperature of hot junction, till the temperature becomes equal to be neutral temperature of the hot junction. As the temperature is increased beyond the neutral temperature, thermo e.m.f. starts decreasing.

Q. 36. Write the expression connecting the thermoelectric e.m.f. of a thermocouple with the temperature difference of its cold and hot junctions.

Ans. The relation between thermo e.m.f. (E) and temperature difference (θ) between the cold and the hot junction is given as

$$E = \alpha\theta + \beta\theta^2$$

Q. 37. Name a few metals, which have (i) positive Thomson coefficient and (ii) negative Thomson coefficient.

Ans. (*i*) For copper, antimony, silver and Zinc, Thomson coefficient is positive

(*ii*) For iron, cobalt, bismuth and platinum, Thomson coefficient is negative.

Q. 38. What is the cause of production of thermo e.m.f. in the thermocouple?

Ans. In a thermocouple, heat is absorbed at the hot junction, while it is rejected at the cold junction. The production of thermo e.m.f. in a thermocouple is the result of conservation of the net heat absorbed in the thermocouple into electric energy. In other words, the thermoelectric effect obeys the law of conservation of energy.

Q. 39. Heat is produced at a junction of two metals, when a current passes through. When the direction of current is reversed, heat is absorbed at the junction (i.e. the junction

gets cooler) Is the usual formula (I^2R = Power dissipated as heat) applicable for this situation. If not why not?

Ans. No, the formula is not applicable to the situation, when on reversing the direction of current, the heat is absorbed at the hot junction. It is because Joule's heating effect of current and reversible Seebeck effect are different from each other.

Q. 40. How does thermoelectric series help to predict the direction of flow of current in a thermocouple?

Ans. It helps to know the direction in which current will flow, when a thermocouple is formed with the wires of any two metals in the series. The direction of current will be from a metal occurring earlier in this series to the metal occurring latter in the series through the cold junction.

Q. 41. Why do we generally prefer Sb-Bi thermocouple?

Ans. The metals Sb and Bi are at the two extreme ends of the thermoelectric series and hence for the given temperature of cold and hot junctions, the thermo e.m.f. produced is maximum. It is because more the metals are separated in the series, the greater will be the thermo-e.m.f. produced.

Q. 42. What is a thermopile?

Ans. It is a combination of a large number of thermocouples in series. As such, it is able to detect the heat radiation and to note the small variation or difference in temperature.

8

The Mechanical Equivalent of Heat

8.1 DESCRIPTION OF THE CALLENDER-AND-BARNES CALORIMETER

A heating coil is mounted axially along a horizontal glass tube. This glass tube is further surrounded by a glass jacket to minimise convection of heat. The coil is made of manganin or nichrome. Small length brass tubes are jointed to the two ends of the glass tube by sealing wax. The ends of the heating coil are brought out for external electrical connection by means of two screws. The free ends of the brass tubes are connected with hollow iron bases which have three extra openings i.e. two vertical and one horizontal. The horizontal openings are used for inlet and outlet of water. In one of the vertical openings on both ends of the glass tube a thermometer is inserted through rubber stopper. The other vertical opening on both sides are used to remove any air bubble which might have crept in while flowing water from the tank.

The water reservoir is a small metal vessel having three openings at the bottom. One of the openings is connected to the tap, the middle one to the sink, and the other to the inlet end of the Callender-and-Barnes calorimeter. The height of the reservoir is adjusted and water is allowed to flow through the tube at a constant pressure.

The outlet end of the calorimeter is connected to a small glass tube having a nozzel at the free end by means of a rubber tube. The rate of flow of water from the nozzle is controlled by means of the reservoir attached to the input end. The temperature of the inlet and outlet water are given by the respective thermometers. T_1 and T_2.

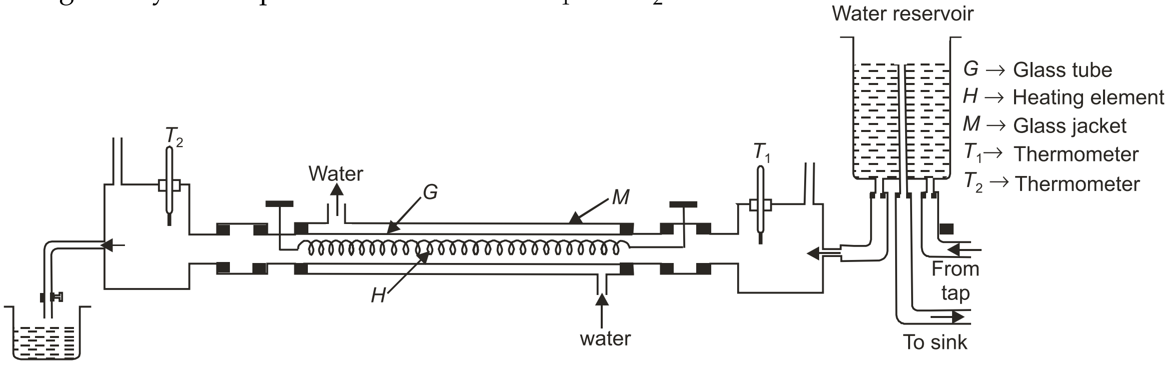


Fig. 8.1

Theory: When a steady electric current flows through the heating coil and a steady flow of water is maintained through the tube, the temperatures at all parts of the apparatus become steady. Under such steady-state conditions, the amount of electrical energy supplied during a known time interval is consumed in heating the amount of water which flows through the tube during the same interval and a small amount of heat is lost by radiation etc., to the surroundings during that interval.

Let the current flowing through the heating coil $= I_1$ amps
the potential difference between the ends of the coil $= V_1$ volts
the rate of flow of water through the tube $= m_1$ gm/sec
the temperature of the inlet water $= \theta_1$°C
the temperature of the outlet water $= \theta_2$°C
and the mean specific heat of water between the temperatures t_1 and t_2 $= s$

Therefore, we can write

$$\frac{V_1 I_1}{J} = m_1 s\,(\theta_2 - \theta_1) + h_1 \quad ...(1)$$

Where J is the mechanical equivalent of heat (also called Joule's equivalent) and h_1 is the amount of thermal leakage per second from the surface of the tube due to radiation etc.

If V_1, I_1, and m_1 are changed to V_2, I_2, and m_2 while keeping the temperature rise unaltered, then for the same surrounding temperature we can write

$$\frac{V_2 I_2}{J} = m_2 s\,(\theta_2 - \theta_1) + h_2 \quad ...(2)$$

Subtracting Eq. (1) from Eq.(2), we obtain

$$\frac{V_2 I_2 - V_1 I_1}{J} = s\,(m_2 - m_1)\,(\theta_2 - \theta_1) + (h_2 - h_1)$$

For all practical purposes, we may consider $h_1 = h_2$

$$\boxed{J = \frac{V_2 I_2 - V_1 I_1}{s\,(m_2 - m_1)\,(\theta_2 - \theta_1)}}$$

Thus, by measuring V_1, V_2, I_1, I_2, m_1, m_2, θ_1 and θ_2, and knowing s, J can be determined in Joules/Calorie.

8.2 OBJECT

To determine the Mechanical Equivalent of heat (J) by the Callender and Barnes method.

Apparatus used: A Callender and Barne's calorimeter, AC mains with a step down transformer, an AC Ammeter and an AC Voltmeter, switch, a rheostat, a stop watch, a measuring jar and 2 thermometers.

Formula used: $J = (E_2C_2 - E_1\,C_1)/(m_1 - m_2)\,(\theta_2 - \theta_1)\,s$ for water $S = 1.0$ Cal/gm °C.

Procedure:

1. Connect the apparatus as shown in the Fig. 8.2.
2. Adjust the tap and the water reservoir till the rate of flow of water through the tube is about (one) c.c per second.

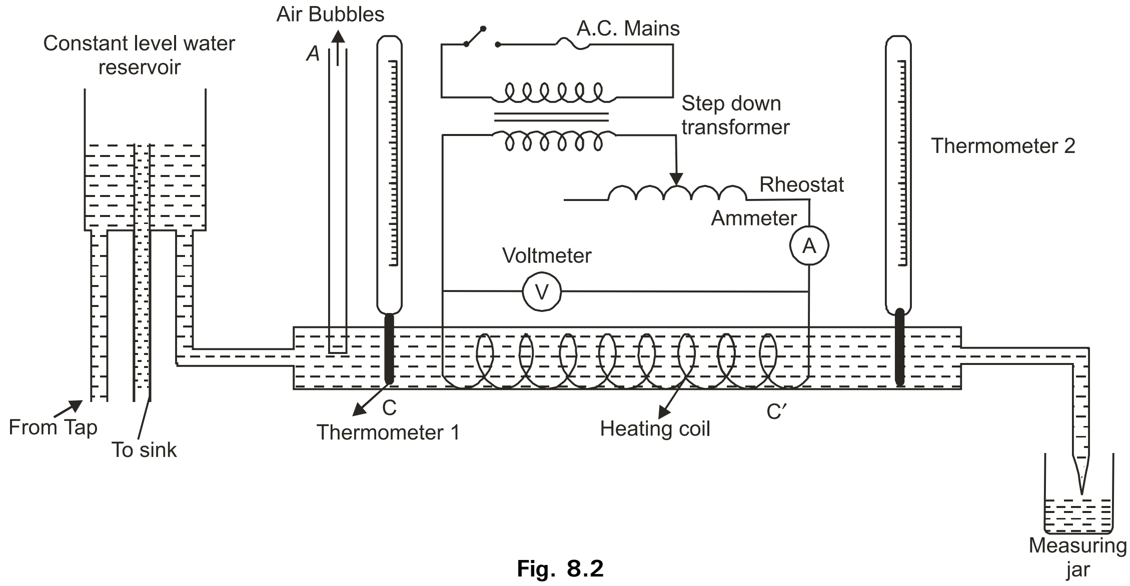


Fig. 8.2

3. Switch on the current and regulate the rheostat so that the current passing is about 2 amperes.
4. As soon as the temperature of the heated water going out becomes steady. Note the temperature of the two thermometers. Note the ammeter and the voltmeter readings.
5. Measure the rate of flow of water at this moment with the help of measuring Jar.
6. Change the rate of flow of water by varying the height of the reservoir and vary the electric current until the two thermometers again indicate their previous readings. Note the new readings of the ammeter and the voltmeter and measure the new rate of flow of water.

Observation:

Temperature of the cold water (inlet end) = θ_1 _ _ _ _ _ ºC
Temperature of the hot water (exit end) = θ_2 = _ _ _ _ _ ºC

	E (in volts)	*C (in amps)*	*Amount of flow of water per minute unit*			
			I	II	III	Mean
I Case						
II Case						

Result: The value of *J* is found to be = ergs/cal. (C.G.S. units)
= Joule/cal. (M.K. S. units)

Precautions:

1. The rate of flow of water in the tube should be uniform. To ensure this a number of measurements for the rate of out flow of water should be made.
2. Heating of the water should be uniform throughout tube.
3. Thermometers should be very sensitive.

8.3 VIVA-VOCE

Q. 1. Define mechanical equivalent of heat?

Ans. The mechanical equivalent of heat is defined as the amount of work done in order to produce a unit calorie of heat.

Q. 2. Why do you call it by the letter *J*?

Ans. It is represented by *J*, which is the first letter in the name Joule. James Prescot Joule was the first to determine the value of the ratio of work done to the heat produced.

Q. 3. What are the units of *J*?

Ans. In the C.G.S system, the units of *J* are ergs/calorie.

Q. 4. What is standard value of J?

Ans. It is 4.1852×10^7 erg per calorie or 4.1852 *J*/cal.

Q. 5. What is meant by mechanical equivalent of heat?

Ans. The mechanical equivalent of heat *J* is defined as the constant ratio between mechanical work done '*W*' and corresponding heat produced '*H*' i.e., $J = W/H$.

Q. 6. Why is the heating coil taken in the form of helical form?

Ans. Because greater surface area is exposed to water and it keeps water stirred.

Q. 7. Why should a constant level water tank be used?

Ans. If a constant level water tank is not used for steady flow of water, a steady difference of temperature between the thermometers will not be obtained.

Q. 8. What is Joule's law on Heating Effects of Currents?

Ans. The quantity of heat *H* produced due to a current *C* in a conductor is directly proportional to (i) the square of the current (ii) the resistance *R* and (iii) the time t sec. Thus

$$H = \frac{VCt}{J} = \frac{C^2Rt}{J}$$

Q. 9. In what units are current and voltage used in it?

Ans. The current and voltage have been taken in electromagnetic units.

Q. 10. How do you convert these into practical units?

Ans. The practical unit of voltage is a volt such that 1 volt = 10^8 e.m.u. of potential difference. The practical unit of current is an ampere such that 1 ampere = $\frac{1}{10}$ e.m.u. of current. Thus the heat produced, $H = \frac{Vct}{J} \times 10^7 = \frac{C^2Rt}{4.18} = .24C^2\,Rt$

Q. 11. How much work is done in heating by means of currents?

Ans. If a current of *C* amperes passes in a conductor whose ends are maintained at a potential difference of *V* volts, then in t secs the amount of work done.

$$W = VCt \times 10^7 \text{ ergs}$$

Q. 12. Why is this work done when current passes through a conductor?

Ans. When a potential difference is applied across a conductor, than the electric current is due to a flow of electrons in the interatomic space of the conductor. These electrons experience resistance to their motion in that space and so work has to be done against this resistance. This work done appears as heat.

Q. 13. Upon what factors does the work done depend?

Ans. The work done depends upon the following factors:

(*i*) The number of electrons flowing, i.e., the strength of the current in the conductor.

(*ii*) The resistance of the conductor.

(*iii*) The time for which the current flows in the conductor.

Q. 14. Does the heating effect depend upon the direction of current?

Ans. The heating effect of current does not depend upon the direction of current because it is proportional to the square of the current.

Q. 15. Will the heating effect be different for direct and alternating currents?

Ans. No, the effect is the same with both types of currents because it does not depend upon the direction of current.

Q. 16. What is the heating coil made of?

Ans. The heating coil is made of some resistance wire such as nichrome, constantan or manganin.

Q. 17. Can you give an idea of the resistance of the heater coil?

Ans. The resistance of the heater coil can be calculated by ohm's law by dividing any voltmeter reading with the ammeter reading.

Q. 18. Should the resistance be high or low?

Ans. The resistance of heater coil should be low so that it may take up a large current and the heating may be large, because $H \propto C^2$.

Q. 19. Why is the water not electrolyzed if the naked heating coil is placed in the water?

Ans. Water is a poor conductor of electricity and so all the current passes through the coil and not through water. Thus the water is not electrolyzed.

If ordinary water be used and the potential difference used be more than eight volts, some electrolysis may take place.

Q. 20. Why do you use a step down transformer while using AC?

Ans. A step down transformer is used in order that the apparatus may be used safely without any danger of getting a shock.

Q. 21. Can this apparatus be used for any other purpose?

Ans. Originally this was designed for the study of variation of specific heat with temperature. It can also be used for determining the specific heat of air at constant pressure and of mercury or any other liquid.

Q. 22. What important precaution are taken by you?

Ans.
1. Water should flow at a constant pressure and its motion should be slow and continuous.
2. The rate of flow of water and the two temperatures should be noted only when the steady state has been reached.
3. The current is switched on after the tube is filled up with water.
4. The difference of temperature should be maintained at about 5ºC and sensitive thermometers should be used.
5. The sets of observations are taken for different currents for the same difference of temperature.

Q. 23. What other methods of determining *J* do you know? Discuss their relative merits and demerits?

Ans. '*J*' by Searle's Friction Cone method.

Q. 24. What would happen if the flow of water is not steady?

Ans. The difference of temperature will not remain steady.

7
Sound

7.1 SPEED OF TRANSVERSE WAVE IN STRETCHED STRING

A string means a wire or a fibre which has a uniform diameter and is perfectly flexible *i.e.* which has no rigidity. In practice, a thin wire fulfills these requirements approximately.

The speed of transverse wave in a flexible stretched string depends upon the tension in the string and the mass per unit length of the string. Mathematically, the speed v is given by

$$v = \sqrt{\frac{T}{m}}$$

Where T is the tension in the string and m is the mass per unit length of the string (not the mass of the whole string).

If r be the radius of the string and d the density of the material of the string, then

$$m = \text{volume per unit length} \times \text{density} = \pi r^2 \times 1 \times d = \pi r^2 d$$

Then the speed of transverse wave is

$$v = \sqrt{\frac{T}{\pi r^2 d}}$$

7.2 VIBRATIONS OF STRETCHED STRING

When a wire clamped to rigid supports at its ends is plucked in the middle, transverse progressive waves travel towards each end of the wire. The speed of these waves is

$$v = \sqrt{\frac{T}{m}} \quad \text{...(i)}$$

Where T is the tension in the wire and m is the mass per unit length of the wire. These waves are reflected at the ends of the wire. By the superposition of the incident and the reflected waves, transverse stationary waves are set up in the wire. Since the ends of the wire are clamped, there is a node N at each end and a antinode A in the middle (Fig. 7.1).

We know that the distance between two consecutive nodes is $\frac{\lambda}{2}$, where λ is wavelength. Hence if l be the length of the wire between the clamped ends, then

$$l = \frac{\lambda}{2}$$

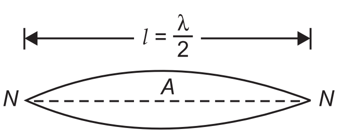


Fig. 7.1

or $$\lambda = 2l$$

If n be the frequency of vibration of the wire, then $n = \frac{v}{\lambda} = \frac{v}{2l}$

Substituting the value of v from Eq. (i) we have $n = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)}$ ...(ii)

This is the frequency of the note emitted by the wire.

It is seen from Eq. (ii) that the frequency of the sound emitted from a stretched string can be changed in two ways by changing the length of the string or by changing the tension in the string. In sitar and violin the frequencies of the notes are adjusted by tightening or loosening the pegs of the wires.

7.3 FUNDAMENTAL AND OVERTONES OF A STRING

When a stretched wire is plucked in the middle, the wire usually vibrates in a single segment Fig. 7.2(a). At the ends of the wire are nodes (*N*) and is the middle an antinode (*A*). In this condition, the node emitted from the wire is called the "fundamental tone". If l be the length of the wire, and λ_1 be the wavelength in this case, then

$$l = \frac{\lambda_1}{2}$$

or $$\lambda_1 = 2l$$

If n_1 be the frequency of vibration of the wire and v the speed of the wave in the wire, then

$$n_1 = \frac{v}{\lambda_1} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

This is the fundamental frequency of the wire.

We can make the wire vibrate in more than one segment. If we touch the middle-point of the wire by a feather, and pluck it at one-fourth of its length from an end, then the wire vibrates in two segments Fig. 7.2(*b*). In this case, in addition at the ends of the wire, there will be a node (*N*) at the middle-point also, and in between these three nodes there will be two antinodes (*A*). Therefore, if λ_2 be the wavelength in this case, then

$$l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \frac{2\lambda_2}{2}$$

or $$\lambda_2 = \frac{2l}{2}$$

If the frequency of the wire be now n_2, then

$$n_2 = \frac{v}{\lambda_2} = \frac{2v}{2l} = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2n_1$$

that is, in this case the frequency of the tone emitted from wire is twice the frequency of the fundamental tone. This tone is called the 'first overtone'.

Similarly, if the wire vibrates in three segments Fig. 7.2(*c*) and the wavelength in this case be λ_3, then

$$l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} = \frac{3\lambda_3}{2}$$

or $$l_3 = \frac{2l}{3}$$

If the frequency of the wire be n_3, then

$$n_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3n_1$$

that is, in this case the frequency of the emitted tone is three times the frequency of the fundamental tone. This tone is called 'second overtone'.

Similarly, if the wire is made to vibrate in four, five .... segments then still higher overtones can be produced. If the wire vibrates in p segments, then its frequency is given by

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}}$$

Thus, the frequencies of the fundamental tone and the overtones of a stretched string have the following relationship:

$$n_1 : n_2 : n_3 : ...... = 1 : 2 : 3 : ......$$

These frequencies are in a harmonic series. Hence these tones are also called 'harmonics'. The fundamental tone (n_1) is the first harmonic, the first overtone (n_2) is the second harmonic, the second overtone (n_3) is the third harmonic etc. The tones of frequencies n_1, n_3, n_5 ..... are the odd harmonics and the tones of frequencies n_2, n_4, n_6....... are the 'even harmonics'. Clearly, a stretched string gives both even and odd harmonics.

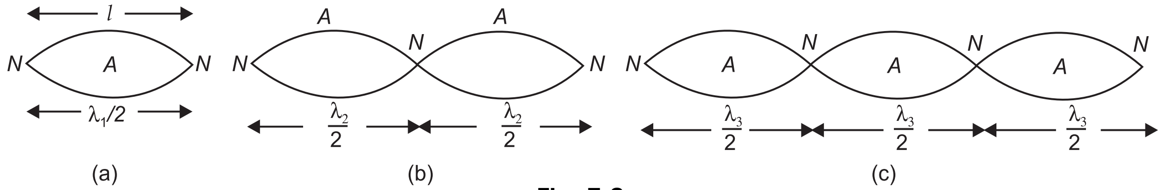


Fig. 7.2

7.4 SONOMETER

It is the simplest apparatus for demonstrating the vibrations of a stretched string. It consists of a hollow wooden box about 1 meter long which is called the 'sound board'. A thin wire is stretched over the sound-board. One end of the wire is fastened to a peg A at the edge of the sound-board and the other end passes over a frictionless pulley P and carries a hanger upon which weights can be placed. These weights produce tension in the wire and press it against two bridges B_1 and B_2. One of these bridges is fixed and the other is movable. The vibrating length of the wire can be changed by changing the position of the movable bridges. The wall of the sound-board contains holes so that the air inside the sound-board remains in contact with the air outside. When the wire vibrates, then these vibrations reach (through the bridges) the upper surface of the sound-board and the air inside it. Along with it, the air outside the sound-board also begins to vibrate and a loud sound is heard.

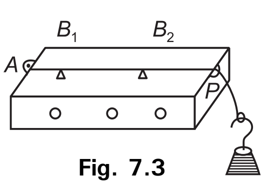


Fig. 7.3

When the sonometer wire is plucked at its middle-point, it vibrates in its fundamental mode with a natural frequency '*n*' is given by $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$.

Here *l* is the length of the wire between the bridges, *T* is the tension in the wire and *m* is the mass per unit length of the wire. If *r* be the radius of the wire, *d* the density of the material of the wire and *M* the mass of the weights suspended from the wire, then

$$m = \pi r^2 d \text{ and } T = Mg$$

$$\therefore \qquad n = \frac{1}{2l}\sqrt{\frac{Mg}{\pi r^2 d}}$$

7.5 OBJECT

To determine the frequency of A.C. mains by using a sonometer and a horse-shoe magnet.

Apparatus: A sonometer, a step-down transformer, a choke, weights, a meter scale and a scale pan, a horse-shoe magnet, a wire of non-magnetic material (brass or copper wire), a physical balance, a weight-box.

Formula used: The frequency of A.C. mains is given by the following formula.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Where l = length of the sonometer wire between the two bridges when it is thrown into resonant vibrations.

T = tension applied to the wire.

m = mass per unit length of the wire.

Description of apparatus: A sonometer consists of a wooden box *AB* about 1 metre long. It also carries a wire of uniform cross-section and made of non-magnetic material usually brass. One end of this wire is fixed to a peg at one end of the box. This wire after passing over a pulley at the other end of the box carries a hanger at the other end. Tension is produced in the wire by placing suitable load on this hanger. There are three knife-edge-bridges over the box. Two of them are fixed near the ends of the box while the third one, can be slided along the length of the wire supporting it (Some times only two knife-edge-bridges is provided in this case, one is fixed and other is slide to get maximum vibration). Its position can be read on a scale fitted along the length of the wire. The vibrations of the wire alone can not produce audible sound. But the sound box helps in making this sound louder. When wire vibrates, these vibrations are communicated to the box and the enclosed air in it. Since the box has a large surface and volume it produces sufficient vibrations in air to make it audible.

A permanent horse-shoe magnet is mounted vertically in the middle of the wire with wire passing between its poles. The magnet produces a magnetic field in the horizontal plane and perpendicular to the length of the wire. When the alternating current from mains after being stepped down to 6 or 9 volt is passed through the wire, it begins to vibrate in vertical plane. By adjusting the position of the bridge resonance can be obtained.

Theory: When transverse waves are excited in a stretched wire the bridges act as rigid reflectors of these waves. As a result of this the length of the wire between two bridges becomes a bound medium with waves reflected at both ends. Thus stationary waves are formed with bridges as nodes. Therefore in the fundamental mode, when wire vibrates in one loop, we have

$$\frac{\lambda}{2} = l$$

Where l is the distance between bridges and λ is the wave-length of transverse waves through the string. We know that if the, elastic forces are negligible compared to tension, the velocity of transverse waves in the string is given by

$$v = \sqrt{\frac{T}{m}}$$

where T is the tension and m is mass per unit length of the wire. Therefore, the natural frequency (fundamental mode) of the wire is given by

$$n = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

The frequency of the wire can be changed by varying tension T, or length l.

Now when the wire carrying current is placed in a magnetic field perpendicular to its length, the wire experiences a magnetic force whose direction is perpendicular to both the wire as well as the direction of the magnetic field. Thus due to orientations of field and wire, the wire in this case experiences a force in the vertical direction with the sense given by Fleming's left hand rule. Since in the experiment alternating current is being passed through the wire, it will experience an upward force in one half cycle and downward force in next half cycle. Thus the wire gets impulses alternately in opposite directions at the frequency of the current, and consequently it begins to execute forced transverse vibrations with the frequency f of the alternating current. Now if the distance between bridges is so adjusted that the natural frequency of vibrations 'n' of the wire becomes equal to that of the alternating current, resonance will take place, and the wire will begin to vibrate with large amplitude. In this case $f = n$. Hence

$$f = n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

From this the frequency of A.C. mains can be calculated.

Procedure:

1. Arrange the apparatus as shown in Fig. 7.3.
2. Put some weights on the pan and the magnet on the board between the bridges in such a position as to produce magnetic field at right angles to the wire.
3. Connect the primary of the step-down transformer to A.C. mains.
4. Now vary the position of the bridges slowly and symmetrically with respect to the magnet till a stage is reached when the wire vibrates with maximum amplitude. This is the position of resonance. Measure the distance between bridges. Repeat this step 3 or 4 times to find mean value of l.
5. Repeat above steps with load on the hanger increasing in steps of 100 gm till maximum allowable limit is reached. Corresponding to each load find mean l.
6. Repeat the experiment with load decreased in the same steps in which it was increased.
7. From readings with increasing and decreasing load find mean value of l corresponding to each load.
8. Weigh the specimen wire, measure its length and hence calculate its linear density.

Observations:

1. Measurement of '*T*' and '*l*'
   Mass of the pan =

S.No.	*Total load (including hanger) M gms.*	*Length in resonance l 'cm'*		*Mean length*
		Load increasing	*Load decreasing*	
1. 2. 3. 4. 5. 6.				

2. Measurement of '*m*'—
   (*i*) Mass of the specimen wire = gm
   (*ii*) Length of the specimen wire = cm

Calculations: Linear density of the wire *m* = gm/cm

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Results: Frequency of the AC mains is found to be = cycles/sec or Hertz.
Standard value = 50 Hz
Percentage error = ... %

Precautions and sources of error:

1. The wire from which the pan is suspended should not be in contact with any surface.
2. Use choke to limit the current or wire may burn out.
3. The wire may be uniform and free from kinks and joints.

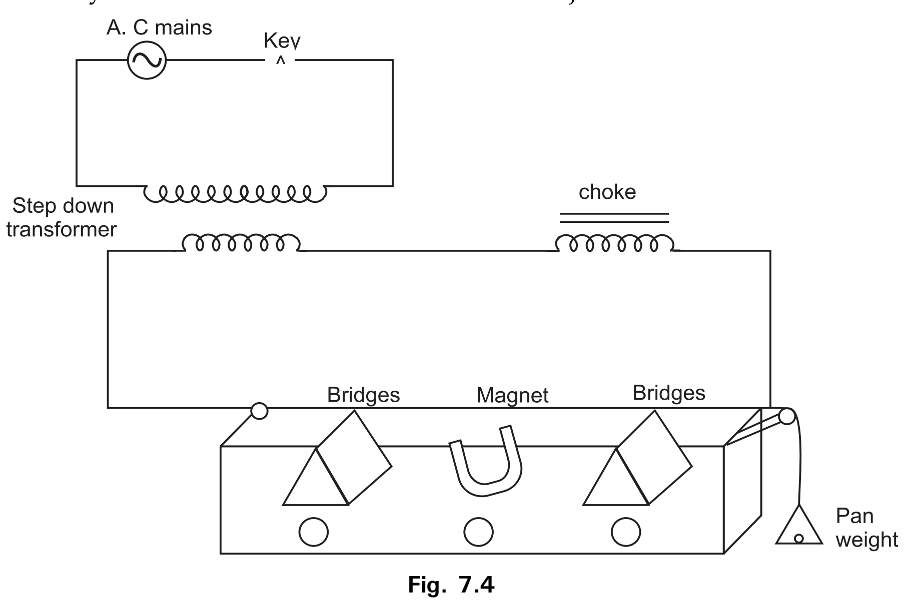


Fig. 7.4

4. The magnetic field should be at center of vibrating loop and must be perpendicular to the length of the wire.
5. The material of the sonometer wire should be non magnetic.
6. The bridges used should give sharp edges to get the well defined nodes.
7. The weights should be removed from the wire otherwise the wire may develop elastic fatigue.
8. In order that the tension in the cord may be exactly equal to the weight suspended, there should be no friction at the pulley.

7.6 OBJECT

To determine the frequency of A.C. mains or of an electric vibrator, by Melde's experiment, using:

(*i*) Transverse arrangement
(*ii*) Longitudinal arrangement

Apparatus used: Electric vibrator, thread and pulley, chemical balance and metre scale.

Formula used:

(*i*) For the transverse arrangement, the frequency n of the fork is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{m}}$$

Where l = length of the thread in the fundamental vibration.
T = tension applied to thread.
M = total mass suspended.
m = mass per unit length of thread.

(*ii*) For the longitudinal arrangement, the frequency of electric vibrator is given by

$$n = \frac{1}{l}\sqrt{\frac{T}{m}} = \frac{1}{l}\sqrt{\frac{Mg}{m}}$$

Where the symbols have usual meaning.

Description of the apparatus: An electric vibrator consists of a solenoid whose coil is connected to A.C. mains. The circuit includes a high resistance in the form of an electric bulb as shown in Fig. 7.5. A soft iron rod AB is placed along the axis of the solenoid, clamped near the end A with two screws X and Y while the end B is free to move. The rod is placed between the pole pieces of a permanent magnet NS. One end of the thread is attached to the end B and the other passes over a frictionless pulley and carries a weight.

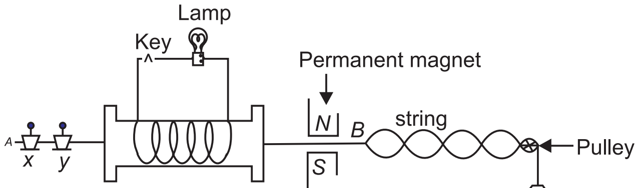


Fig. 7.5

When an alternating current is passed in the coil of the solenoid, it produces an alternating magnetic field along the axis. The rod AB gets magnetised with its polarity changing with the same frequency as that of the alternating current. The rod AB vibrates n times per second due to interaction of the magnetised rod with the permanent magnet.

Procedure: Transverse arrangement

1. Take a uniform thread and attach its one end to the point B of the rod and the other to a light pan by passing it over a frictionless pulley.
2. Connect the A.C. mains as shown in Fig. 7.4.
3. Place the vibrator in the transverse position.
4. The vibrations of maximum amplitude are obtained either by adding weights to the pan slowly in steps or by puting some mass M in the pan and adjusting the length of the thread by moving the vibrator.
5. Note the number of loops p formed in the length L of the thread. This gives the value of l as $l = \frac{L}{p}$.
6. Repeat the above procedure for different loops.

Longitudinal arrangement: In this case the vibrator is adjusted such that the motion of the rod is in the same direction as the length of the thread. The procedure remaining the same as described in case of transverse arrangement.

Observations:

1. Mass of the pan =
   Mass of the thread =
   Length of the thread =
2. Transverse arrangement.
   Table for the determination of T and l.

S.No.	*Tension T applied*			*No. of loops (p)*	*corresponding length of the thread L meter*	*Length l for one loop (L/p)*	*Mean l meter*
	weight placed in pan	*weight of pan*	*Total tension*				

3. Longitudinal arrangement.
   Table for the determination of T and l.

S.No.	*Tension T applied*			*No. of loops (p)*	*corresponding length of the thread L meter*	*Length l for one loop (L/p)*	*Mean l meter*
	weight placed in pan	*weight of pan*	*Total tension*				

Calculations: Mass per unit length of thread $m = \frac{\text{Mass}}{\text{Length}}$ = ...

In transverse arrangement.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Mg}{m}} = \text{...... cycles/sec.}$$

Similarly calculate n from the other sets of observations.

Mean n = ... cycles/sec.

In longitudinal arrangement

$$n = \frac{1}{l}\sqrt{\frac{Mg}{m}}$$

$$= \text{...... cycles/sec.}$$

Similarly calculate n from the other set of observations.

Mean n = ... cycles/sec.

Result: The frequency of A.C. mains, using.

1. Transverse arrangement = ... cycles/sec.
2. Longitudinal arrangement = ... cycles/sec.

Standard result: Frequency of A.C. mains = ... cycles/sec.

Percentage error = ... %

Sources of error and precautions:

1. Pulley should be frictionless.
2. The thread should be then, uniform and inextensible.
3. Weight of the scale pan should be added.
4. The loops formed in the thread should appear stationary.
5. Do not put too much load in the pan.

7.7 VIVA-VOCE

Q. 1. What type of vibrations are produced in the sonometer wire and the surrounding air?

Ans. In the wire transverse vibrations are produced and in the surrounding air longitudinal progressive waves are produced.

Q. 2. How are stationary waves produced in the wire?

Ans. The transverse waves produced in sonometer wire are reflected from the bridges. The two waves superpose over each other and stationary waves are produced.

Q. 3. What do you understand by resonance?

Ans. In case of forced or maintained vibrations, when the frequencies of driver and driven are same then amplitude of vibration of driven becomes large. This phenomenon is called resonance.

Q. 4. Is there any difference between frequency and pitch?

Ans. Frequency is the number of vibrations made by the source in one second while pitch is the physical characteristics of sound which depends upon its frequency.

Q. 5. On what factors does the sharpness or flatness of resonance depend?

Ans. It depends only on natural frequency.

Q. 6. What are the positions of nodes and antinodes on sonometer wire?

Ans. Nodes at the bridges and antinodes at the middle between the two knife bridges.

Q. 7. On what factor does the frequency of vibration of a sonometer wire depend?

Ans. The frequency of the wire can be changed by varying tension T, or length l.

Q. 8. What are the laws of vibrations of strings?

Ans. Strings vibrates according to $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Q. 9. What are the requisites of sonometer wire?

Ans. The wire (*i*) should have uniform linear density m, (*ii*) should not change in length during vibration and (*iii*) should be flexible. A steel or brass wire serves the purpose best.

Q. 10. What is the function of the sonometer board?

Ans. The board is hollow and contains air inside. When the vibrations of wire take place, the energy of vibrations is communicated to the board and from there to the enclosed air. Due to the forced vibrations of this large mass of air loudness of sound is increased. The holes drilled on the sides of the board establish the communication of inside air with external air.

Q. 11. Why are bridges provided on the board?

Ans. The bridges limit the length of vibrating wire. Reflection of transverse waves on the string takes place from these bridges and the stationary waves are formed.

Q. 12. How does the friction affect the result?

Ans. Friction reduces the tension applied to the wire i.e. the tension in wire becomes less than the load suspended from the hanger as a result the calculated values are higher than the actual frequency.

Q. 13. In sonometer experiment is the resonance sharp or flat?

Ans. It is sharp. A slight displacement of bridges causes a considerable fall in amplitude of vibration.

Q. 14. What do you mean by A.C. mains?

Ans. An electric current that reverses its direction with a constant frequency (f). If a graph of the current against time has the form of a sine wave, the current is said to be sinusoidal.

Q. 15. What do you mean by frequency of A.C. mains?

Ans. A current which changes its direction of flow *i.e*. continuously varying from zero to a maximum value and then again to zero and also reversing its direction at fixed interval of time.

Q. 16. What is the frequency of your A.C. mains? What does it represent?

Ans. The number of times the current changes its direction in each second is called the frequency of A.C. mains. It's value is 50 cycles per second.

Q. 17. Does direct current also have any frequency?

Ans. No, it does not change its direction.

Q. 18. In case of non-magnetic wire why does it vibrate? When does the wire resonate?

Ans. It vibrates according to Fleming left hand rule.

Q. 19. Can a rubber string be used in place of wire?

Ans. No, the rubber string will not continue vibrating long because it is not sufficiently rigid.

Q. 20. Why should the magnet be placed with its poles in a line perpendicular to the length of wire ?

Ans. To fulfil the condition for vibration according to Fleming left hand rule.

Q. 21. Why do you use a transformer here? Can't you apply the A.C. directly.

Ans. The transformer is used to step down the A.C. voltage to a small value of about 6-9 volts. This ensures that no high current flows through the sonometer wire and heats it up. The A.C. mains is not directly connected to the wire as it is dangerous for human body, and may also cause a high current to flow through the wire.

Q. 22. What is the construction of your transformer?

Ans. A device for transferring electrical energy from one alternating current circuit to another with a change of voltage, current, phase, or impedance. It consists of a primary winding of N_p turns magnetically linked by a ferromagnetic core or by proximity to the secondary winding of N_s turns. The turns ratio $\frac{N_s}{N_p}$ is approximately equal to $\frac{V_s}{V_p}$ and to $\frac{I_p}{I_s}$, where V_p and I_p are the voltage and current fed to the primary winding and V_s and I_s are the voltage and current induced in the secondary winding assuming that there are no power losses in the core.

Q. 23. In above experiment can't we use an iron wire?

Ans. We can't use an iron wire because in this case wire is attracted by magnet and hence wire does not vibrate.

Q. 24. In this experiment will the frequency of sonometer wire change by changing the distance between the bridges.

Ans. No, the frequency of vibration of the sonomater wire will not change by changing the distance between the bridges because the wire is executing forced vibrations with the frequency of the mains.

Q. 25. Then, what is actually changing when the distance between bridges is changed.

Ans. The natural frequency of the wire.

Q. 26. What is the principle, according to which the wire begins to vibrate, when the alternating current is passed through it?

Ans. When a current carrying wire is placed in a magnetic field, it experiences a mechanical force (given by Fleming's left hand rule) which is perpendicular to the direction of current and magnetic field both.

Q. 27. What is Fleming's left hand rule?

Ans. According to this rule if the thumb and the first two fingers of the left hand are arranged mutually perpendicular to each other, and the first finger points in the direction of magnetic field, the second in the direction of current then the thumb indicates the direction of mechanical force.

Q. 28. What do you understand by linear density of wire?

Ans. Mass per unit length is called the linear density of wire.

Q. 29. Why a choke is used?

Ans. To avoid the heating of the wire.

Q. 30. What are the losses in the transformer.

Ans. Eddy current loss, hysteresis losses in the wire, heating losses in the coils themselves.

Q. 31. Why a special type (horse-shoe type) of magnet is used?

Ans. In this type of magnet the magnetic field is radial.

Q. 32. How does the rod vibrate?

Ans. When alternating current is passed through the solenoid, the iron rod is magnetised such that one end is north pole while other end is south pole. When the direction of current is changed, the polarity of rod is also changed. Due to the interaction of this rod with magnetic field of permanent horse-shoe magnet, the rod is alternately pulled to right or left and thus begins to vibrate with frequency of A.C. mains.

Q. 33. What type of vibrations does the rod execute?

Ans. The vibrations are forced vibrations. The rod execute transverse stationary vibrations of the same frequency as that of A.C.

Q. 34. Can you use a brass rod instead of soft iron rod?

Ans. No, because it is non-magnetic.

Q. 35. How is it that by determining the frequency of the rod, you come to know the frequency of A.C. mains?

Ans. Here the rod vibrates with the frequency of A.C. mains.

Q. 36. What is the construction of an electric vibrator?

Ans. It consists of a solenoid in which alternating current is passed. To avoid the heating effect in the coil of solenoid, an electric bulb is connected in series. A rod passes through the solenoid whose one end is fixed while the other is placed in pole pieces of permanent horse shoe magnet.

Q. 37. What are resonant vibrations?

Ans. If the natural frequency of a body coincides with the frequency of the driving force, the former vibrates with a large amplitude. Now the vibrations are called as resonant vibrations.

Q. 38. When does resonance occur?

Ans. When the natural frequency of the rod becomes equal to the frequency of AC mains, resonance occurs.

Q. 39. How do you change the frequency of the rod?

Ans. We can change the frequency of the rod by changing the vibrating lengths of rod outside the clamps.

Q. 40. Can't we send direct current through solenoid?

Ans. No. In this case the ends of the rod will become permanently either N or S pole and will be pulled to one side.

EXERCISE

Q. 1. What is the cause of variation of current in case of A.C.?

Q. 2. For securing resonance, where do you put the magnet and why?

Q. 3. Does a transformer also change the frequency of A.C.? If not, why?

Q. 4. What is the chief source of error in this experiment?

Q. 5. In case of iron wire which arrangement do you use?

Q. 6. Why do you halve the frequency of the wire to obtain the frequency of A.C. mains in this case?

Q. 7. What is the elastic fatigue and elastic limit and how they are related in the experiment.

5

Surface Tension

5.1 SURFACE TENSION

When a small quantity of water is poured on a clean glass plate, it spreads in all directions in the form of a thin film. But when a small quantity of mercury is poured on the glass plate, it takes the form of a spherical drop. Similarly, if a small quantity of water is poured on a greasy glass plate, it also takes the form of small globules like mercury. This shows that the behaviour of liquids is controlled not only by gravitational force (weight) but some other force also acts upon it which depends upon the nature of the surfaces in contact. If the weight of the liquid is negligible then its shape is perfectly spherical. For example rain drops and soap bubbles are perfectly spherical. We know that for a given volume, the surface area of a sphere is least. Hence we may say that the free surface of a liquid has a tendency to contract to a minimum possible area.

The free surface of a liquid behaves as if it is in a state of tension and has a natural tendency to contract and occupy minimum surface area. The behaviour is like that of a stretched elastic or rubber membrane with an important difference that whereas the tension in a membrane increases with stretching the tension in a liquid surface is independent of extension in the area. This property of the liquid is known as surface tension. Various experiments suggest that the surface film exerts a force perpendicular to any line drawn on the surface tangential to it. The surface tension of a liquid can be defined in the following way.

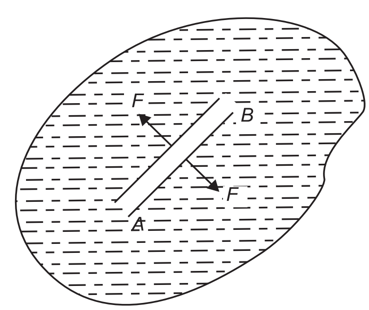


Fig. 5.1

5.2 DEFINITION OF SURFACE TENSION

Let an imaginary line *AB* be drawn in any direction in a liquid surface. The surface on either side of this line exerts a pulling force on the surface on the other side. This force lies in the plane of the surface and is at right angles to the line *AB*. The magnitude of this force per unit length of *AB* is taken as a measure of the surface tension of the liquid. Thus if *F* be the total force acting on either side of the line *AB* of length *l*, than the surface tension is given by

$$T = \frac{F}{l}.$$

If $l = 1$ then $T = F$. Hence, the surface tension of a liquid is defined as the force per unit length in the plane of the liquid surface, acting at right angles on either side of an imaginary line drawn in that surface. Its unit is 'newton/meter' and the dimensions are MT^{-2}.

The value of the surface tension of a liquid depends on the temperature of the liquid, as well as on the medium on the other side of the surface. It decreases with rise in temperature and becomes zero at the critical temperature. For small range of temperature the decrease in surface tension of a liquid is almost linear with rise of temperature.

The value of surface tension for a given liquid also depends upon the medium on outer side of the surface. If the medium is not stated, it is supposed to be air.

5.3 SURFACE ENERGY

Consider a soap film formed in a rectangular framework of wire *PQRS* with a horizontal weightless wire *AB* free to move forward or backward. Due to surface tension the wire *AB* is pulled towards the film. This force acts perpendicular to *AB* and tangential to the film. To keep *AB* in equilibrium a force *F* has to be applied as shown in figure. If *T* is the surface tension of the film, then according to definition.

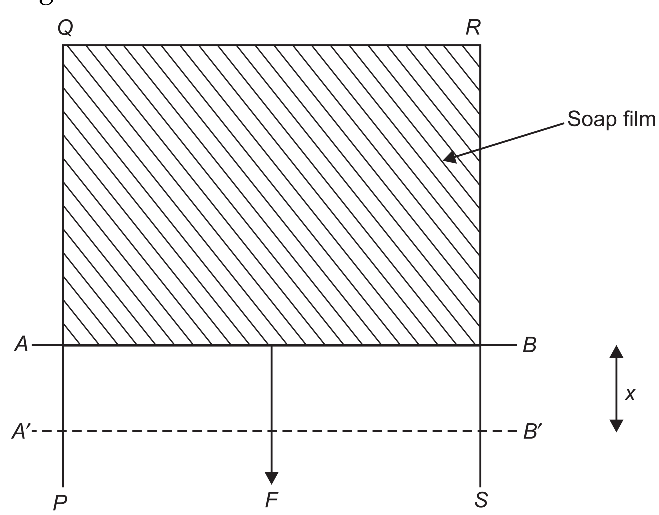


Fig. 5.2

(*i*) the film pulls the wire by a force $2l \times T$ because the film has two surfaces and l is the length of wire *AB*. Thus for equilibrium

$$2l \times T = F$$

or $$T = \frac{F}{2l}$$

if $$2l = 1,\ T = F$$

Now suppose that keeping the temperature constant the wire *AB* is pulled slowly to *A′B′* through a distance x. In this way the film is stretched by area $\Delta A = 2lx$ and work done in this process is

$\therefore$ $$W = Fx$$

Hence $$W = Fx = 2lxT = \Delta A \times T$$

or $$T = \frac{W}{\Delta A}$$

if $\Delta A = 1$, then $$\boxed{T = W}$$

The work done in stretching the surface is stored as the potential energy of the surface so created. When the surface is stretched its temperature falls and it therefore, takes up heat from surrounding to restore its original temperature. If H is the amount of heat energy absorbed per unit area of the new surface, the total or intrinsic energy E per unit area of the surface is given by

$$E = T + H$$

The mechanical part of energy which is numerically equal to surface tension T is also called free surface energy. The surface tension of a liquid is also very sensitive to the presence of even small quantities of impurities on the surface. The surface tension of pure liquid is greater than that of solution but there is no simple law for its variation with concentration.

5.4 MOLECULAR THEORY OF SURFACE TENSION

Laplace explained the surface tension on the basis of molecular theory. The molecules do not attract or repel each other when at large distances. But they attract when at short distances. The force of attraction is said to be cohesive when it is effective between molecules of the same type. But the force of attraction between molecules of different types is called adhesive force. The greatest distance upto which molecules can attract each other is called the molecular range or the range of molecular attraction. It is of the order of 10^{-7} cm. If we draw a sphere of radius equal to molecular range with a molecule as centre, then this molecule attracts only all those molecules which fall inside this sphere. This sphere is called the sphere of influence or the sphere of molecular attraction.

When a molecule is well inside the liquid it is attracted in all directions with equal force and resultant force, on it is zero and it behaves like a free molecule. If the molecule is very close to the surface of liquid, its sphere of influence is partly inside the liquid and partly outside. Therefore the number of liquid molecules pulling it below is greater than the number of those in the vapour attracting it up. Such molecules experience a net force towards the interior of the liquid and perpendicular to the surface. The molecules which are situated at the surface experience a maximum downward force due to molecular attraction. The liquid layer between two planes FF' and SS' having thickness equal to molecular range is called the surface film. When the surface area of a liquid is increased, more molecules of the liquid are to be brought to surface. While passing through surface film each molecule experiences a net inward force due to cohesion. Mechanical work has to be done to overcome this force. This work is stored in the surface molecules in the form of potential energy. Thus potential energy of molecules in surface film is greater than that in the interior of the liquid. It is a fundamental property of every mechanical system to acquire a stable equilibrium in a state in which its potential energy is minimum. Therefore, to

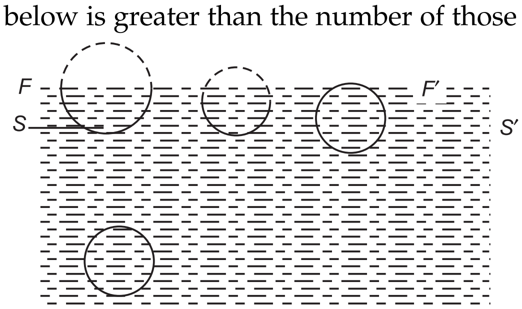


Fig. 5.3

minimize their potential energy the molecules have a tendency to go into the interior of the liquid as a result of which the surface has a tendency to contract and acquire a minimum area. This tendency is exhibited as surface tension.

The potential energy of the molecules in the unit area of the surface film equals the surface energy.

5.5 SHAPE OF LIQUID MENISCUS IN A GLASS TUBE

When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The nature of the curvature (concave or convex) depends upon the relative magnitudes of the cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and those of the solid.

In Fig. 5.4(*a*), water is shown to be in contact with the wall of a glass tube. Let us consider a molecule *A* on the water surface near the glass. This molecule is acted upon by two forces of attraction:

(*i*) The resultant adhesive force *P*, which acts on *A* due to the attraction of glass molecules near *A.* Its direction is perpendicular to the surface of the glass.

(*ii*) The resultant cohesive force *Q*, which acts on *A* due to the attraction of neighbouring water molecules. It acts towards the interior of water.

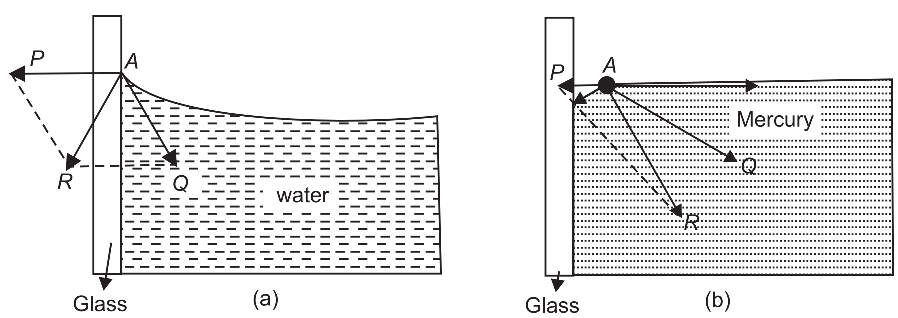


Fig. 5.4

The adhesive force between water molecules and glass molecules is greater than the cohesive force between the molecules of water. Hence, the force *P* is greater than the force *Q*. Their resultant *R* will be directed outward from water. In Fig. 5.4(*b*), mercury is shown to be in contact with the wall of a glass tube. The cohesive force between the molecules of mercury is far greater than the adhesive force between the mercury molecules and the glass molecules. Hence, in this case, the force *Q* will be much greater than the force *P* and their resultant will be directed towards the interior of mercury.

5.6 ANGLE OF CONTACT

When the free surface of a liquid comes in contact of a solid, it becomes curved near the place of contact. The angle inside the liquid between the tangent to the solid surface and the tangent to the liquid surface at the point of contact is called the angle of contact for that pair of solid and liquid.

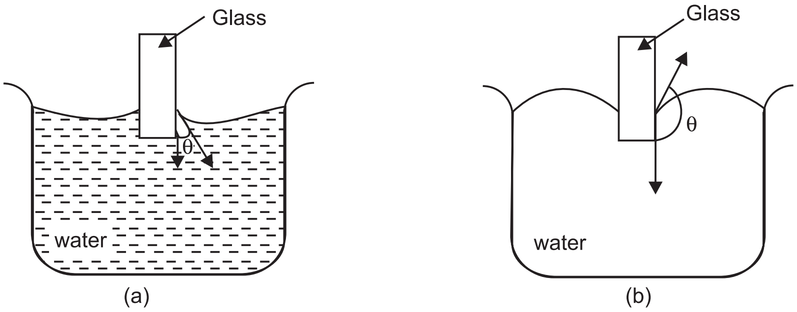


Fig. 5.5

The angle of contact for those liquids which wet the solid is acute. For ordinary water and glass it is about 8º. The liquids which do not wet the solid have obtuse angle of contact. For mercury and glass the angle of contact is 135º. In Fig. 5.5(*a*) and (*b*) are shown the angles of contact θ for water-glass and mercury-glass.

The angle of contact for water and silver is 90º. Hence in a silver vessel the surface of water at the edges also remains horizontal.

5.7 EXCESS OF PRESSURE ON CURVED SURFACE OF LIQUID

The free surface of a liquid is always a horizontal plane. If we consider any molecule on such a surface the resultant force due to surface tension is zero as in Fig. 5.6(*a*). On the other hand if the surface tension acts normally to the surface towards the concave side. Thus for convex meniscus the resultant is directed normally inwards towards the interior of the liquid while for concave meniscus this resultant force is directed normally outwards. As a result of these forces the curved surface has a tendency to contract and become plane. Consequently to maintain a curved liquid surface in equilibrium, there must exist an excess of pressure on the concave side compared to the convex side, which of itself would produce an expansion of the surface.

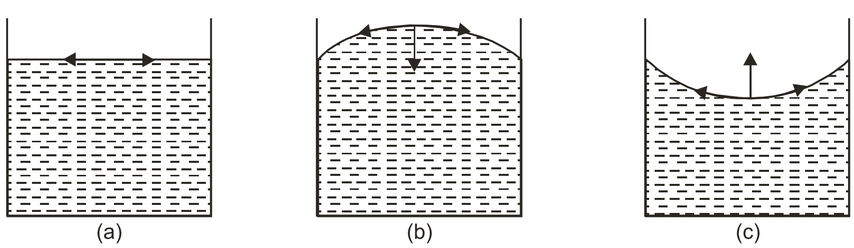


Fig. 5.6

Hence we conclude that there exists an excess of pressure on the concave side of the surface. If the principal radii of curvature of the surface are R_1 and R_2 respectively the magnitude of this excess of pressure on concave side is given by

$$p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(*i*) **Spherical surface:** For spherical surfaces like a liquid drop or air bubble in a liquid there is only one surface and the two principal radii of curvature are equal ($R_1 = R_2 = R$) and we have,

$$p = \frac{2T}{R}$$

But in case of a soap bubble or other spherical films we have two surfaces hence

$$p = \frac{4T}{R}$$

(*ii*) **Cylindrical surface:** In this case one principal radius of curvature is infinite. Therefore

$$p = \frac{T}{R}$$

For cylindrical film since there are two surfaces hence

$$p = \frac{2T}{R}$$

5.8 CAPILLARITY RISE OF LIQUID

When a glass capillary tube open at both ends is dipped vertically in water, the water rises up in the tube to a certain height above the water level outside the tube. The narrower the tube, the higher is the rise of water Fig. 5.7(*a*). On the other hand, if the tube is dipped in mercury, the mercury is depressed below the outside level Fig. 5.7(*b*). The phenomenon of rise or depression of liquid in a capillary tube is called capillarity. The liquids which wet glass (for which the angle of contact is acute) rise up in capillary tube, while those which do not wet glass (for which the angle of contact is obtuse) are depressed down in the capillary.

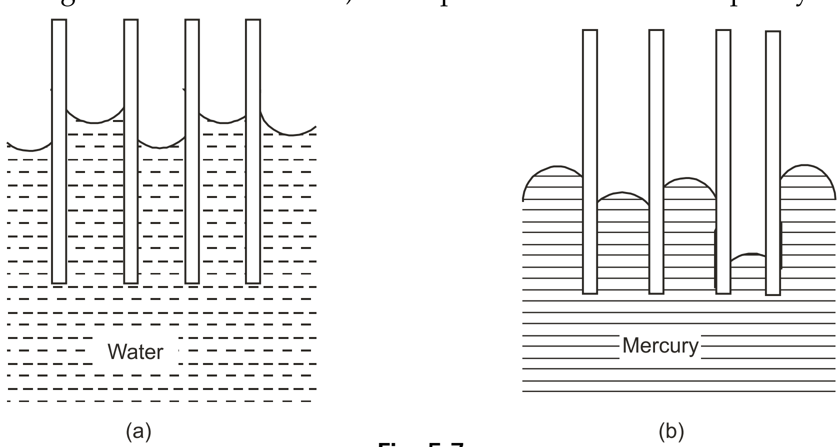


Fig. 5.7

Explanation: The phenomenon of capillarity arises due to the surface tension of liquids. When a capillary tube is dipped in water, the water meniscus inside the tube is concave. The pressure just below the meniscus is less than the pressure just above it by $\frac{2T}{R}$, where T is the surface tension of water and R is the radius of curvature of the meniscus. The pressure on the surface of water is atmospheric pressure P. The pressure just below the 'plane' surface of water outside the tube is also P, but that just below the meniscus inside the tube is $P - \frac{2T}{R}$ Fig. 5.8(*a*). We know that pressure at all points in the same level of water must be the same. Therefore, to make up the deficiency of pressure, $\frac{2T}{R}$, below the meniscus, water begins to flow from outside into the tube. The rising of water in the capillary stops at a certain height 'h' Fig. 5.8(b). In this position the pressure of the water-column of height 'h' becomes equal to $\frac{2T}{R}$, that is $h\rho g = \frac{2T}{R}$ where ρ is the density of water and 'g' is the acceleration due to gravity. If r be the radius of the capillary tube and θ the angle of contact of water-glass, then the radius of curvature R of the meniscus is given by $R = \frac{r}{\cos\theta}$ Fig. 5.8(c).

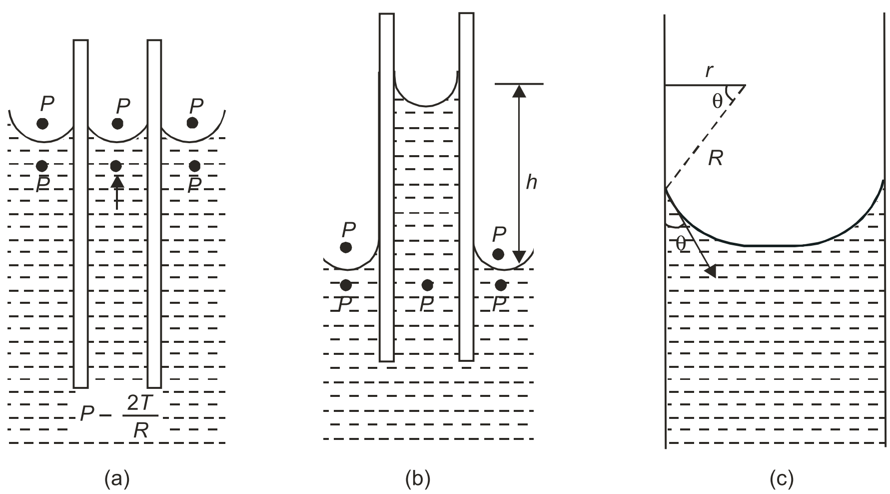


Fig. 5.8

$$\therefore \qquad h\rho g = \frac{2T}{\frac{r}{\cos\theta}}$$

$$h = \frac{2T\cos\theta}{r\rho g}$$

This shows that as r decreases, h increases, that is, narrower the tube, greater is the height to which the liquid rises in the tube.

Rising of liquid in a capillary tube of insufficient length: Suppose a liquid of density ρ and surface tension T rises in a capillary tube to a height 'h' then

$$h\rho g = \frac{2T}{R}$$

Where R is the radius of curvature of the liquid meniscus in the tube. From this we may write

$$hR = \frac{2T}{\rho g} = \text{constant (for a given liquid)}$$

When the length of the tube is greater than h, the liquid rises in the tube to a height so as to satisfy the above equation. But if the length of the tube is less than h, then the liquid rises up to the top of the tube and then spreads out until its radius of curvature R increases to R', such that

$$h'R' = hR = \frac{2T\cos\theta}{\rho g} = \text{const.}$$

It is clear that liquid cannot emerge in the form of a fountain form the upper end of a short capillary tube.

On the other hand if the capillary is not vertical, the liquid rises in the tube to occupy length l such that the vertical height h, of the liquid is still the same as demanded by the formula.

If a tube of non-uniform bore or of any cross-section may be used the rise depends upon the cross-section at the position of the meniscus. If the bore is not circular, the formula for capillary rise is not as simple as Eq. $\left(h = \frac{2T\cos\theta}{r\rho g}\right)$.

5.9 OBJECT

To find the SURFACE TENSION of a liquid (water) by the method of CAPILLARY RISE.

Apparatus used: A capillary tube, petridish with stand, a plane glass strip, a pin, a clamp stand, traveling microscope, reading lens and some plasticine.

Formula used: The surface tension of a liquid is given by the formula.

$$T = r\left(h + \frac{r}{3}\right)\rho g/2 \text{ Newton/meter}$$

Where r = radius of the capillary tube at the liquid meniscus
h = height of the liquid in the capillary tube above the free surface of liquid in the petridish
ρ = density of water ($\rho = 1.00 \times 10^3$ Kg/m^3 for water)

Theory: When glass is dipped into a liquid like water, it becomes wet. When a clean fine bore glass capillary is dipped into such a liquid it is found to rise in it, until the top of the column of water is at a vertical height 'h' above the free surface of the liquid outside the capillary. The reason for this rise is the surface tension, which is due to the attractive forces between the molecules of the liquid. Such forces called cohesive forces try to make the surface of the liquid as small as possible. This is why a drop of liquid is of spherical shape.

Since the surface tension tries to reduce the surface of a liquid we can define it as follows. If an imaginary line of unit length is drawn on the surface of a liquid, then the force on one side of the line in a direction, which is perpendicular to the line and tangential to the surface, is called SURFACE TENSION.

When the liquid is in contact with the glass then on the line of contact the cohesive forces (or surface tension) try to pull the liquid molecules towards the liquid surface and the adhesive forces *i.e.* the forces between the molecules of the glass and the liquid try to pull the liquid molecules towards the glass surface. Equilibrium results when the two forces balance each other. Such equilibrium arises after the water has risen in the capillary to a height of 'h'. This column has weight equal to mg where m is the mass of the water in the column. This balances the upward force due to the surface tension which can be calculated as follows:

The length of the line of contact in the capillary between the surface of the water and the glass is $2\pi r$ where 'r' is the radius of the capillary. As seen from Fig. 5.9, the surface tension T acts in the direction shown and θ is called the angle of contact. The upward component of T is given by $T \cos \theta$ and therefore, recalling that T is the force per unit length, we get the total upward force equal to $2\pi rT\cos\theta$. This must balance the weight mg and we have

$$mg = 2\pi rT \cos \theta$$

For water-glass contact, $\theta = 0°$ and so $\cos \theta = 1$

Therefore, $$mg = 2\pi rT \qquad ...(1)$$

Now $$m = \rho V \qquad ...(2)$$

Where ρ is the density of the liquid and V is the Volume of the column of water

Since radius of the capillary is 'r'

$$V = \pi r^2 h + \text{volume in meniscus (See Fig. 5.9).}$$

and the volume of liquid in the meniscus = volume of cylinder radius 'r' & height 'r' – volume of hemisphere of radius 'r'

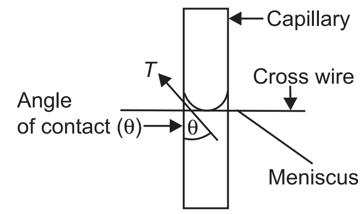


Fig. 5.9: Magnified view of the meniscus in the capillary. The position of the horizonal cross wire is also shown.

i.e. $$\text{volume in meniscus} = \pi r^3 - \left(\frac{2}{3}\right)\pi r^3$$

$$= \left(\frac{1}{3}\right)\pi r^3$$

Therefore, $$V = \pi r^2 h + \left(\frac{1}{3}\right)\pi r^3$$

$$= \pi r^2\left(h + \frac{r}{3}\right) \qquad ...(3)$$

Substituting the value of V from Eq. (3) in Eq. (2) we obtain

$$m = \rho\pi r^2\left(h + \frac{r}{3}\right)$$

and so, Eq. (1) becomes

$$2\pi rT = \rho\pi r^2\left(h + \frac{r}{3}\right)g$$

or $$\boxed{T = \frac{1}{2}\rho gr\left(h + \frac{r}{3}\right)}$$

For water in C.G.S. units, $\rho = 1$ and we finally obtain the formula for a glass capillary dipping in water to be

$$T = \frac{1}{2}gr\left(h + \frac{r}{3}\right) \text{ in C.G.S. units, } i.e. \text{ dynes/c.m. at......°c}$$

Where r = radius of the capillary tube at the liquid meniscus, h = height of the liquid in the capillary tube above the free surface of liquid in the beaker.

Procedure:

1. Mount the capillary on the glass strip using the plasticine and clamp the glass strip so that the capillary is vertical. Pass the pin through the hole in the clamp and secure it so that the tip of the pin is close to the capillary and slightly above its lower end.
2. Fill the petridish with water and place it on the adustable stand just below the capillary and the tip of the pin. Then raise the stand till the capillary dips into the water and the surface of the water just touches the tip of the pin. Observing can ensure that the tip of the pin and its image in the water surface just touch one another. The apparatus will now be in the position as shown in Fig. 5.10.

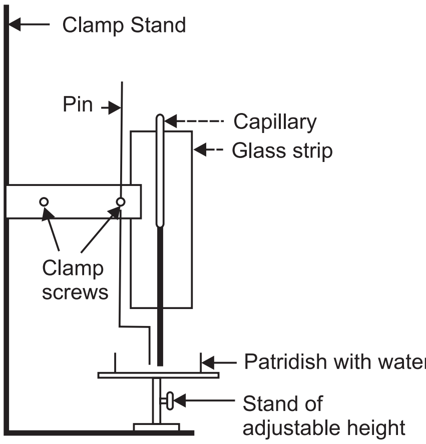


Fig. 5.10: Setup for observing the capillary rise.

3. Level the base of the traveling microscope, so that the upright is vertical. Find the least count of the traveling microscope and the room temperature.
4. Place the microscope in a horizontal position so that its objective is close to the capillary. Focus the crosswire and then move the entire microscope until the capillary is in focus. Now raise (or lower) the microscope until the meniscus is seen. The inverted image (with the curved position above) will be seen in the microscope.
5. Move the microscope and ensure that both the meniscus and the tip of the pin can be seen within the range of the vertical scale.
6. Place the horizontal crosswire so that it is tangential to the meniscus and take the reading on the vertical scale. Repeat it four times.
7. Carefully lower and remove the petridish without disturbing the capillary or the pin. Lower the microscope until it is in front of the tip of the pin. Refocus if necessary, Now take five readings for the tip of the needle, which gives the position where the surface of the water had been earlier. Thus the difference between the readings for the meniscus and the tip of the needle gives the height of the column of water 'h'.
8. The temperature of the liquid is measured and its density ρ at this temperature is noted from the table of constants.
9. To find the radius of the capillary 'r' remove the capillary and the glass strip from the clamp and cut at the point where the meniscus. Place then horizontally so that the tip of the capillary points into the objective of the microscope. Now find the capillary internal diameter by placing the crosswire tangent in turn to opposite sides of the capillary. The position is as shown in Fig. 5.11. The horizontal and vertical diameters must each be determined ten times. This reduces error due to the small value of the diameter and the possibility of an elliptic cross section.

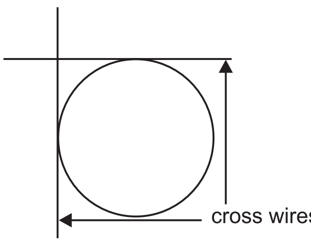


Fig. 5.11: Measurement of capillary radius.

10. The experiment is repeated for different capillary tubes.

Observations: Room temperature = _ _ _ _°C

1. Least count of the traveling microscope = =
2. Reading for the height of the column of water '*h*'

Sl. No	*Reading for Meniscus A (in c.m.)*	*Reading for Tip of needle B (in c.m.)*	*Difference 'h' (in c.m.) h = A – B*
1.			
2.			
3.			
4.			
5.			

Mean value of '*h*' = *c.m.*

3. Reading for diameter of capillary

Sl. No.	*Reading for Horizontal Diameter*			*Reading for Vertical Diameter*			*Diameter (x + y)/2 = D c.m.*	*Radius r = D/2 cm*	*Mean Radius c.m.*
	Left (c.m.)	*Right (c.m.)*	*Difference x (c.m.)*	*Upper (c.m.)*	*Lower (c.m.)*	*Difference y (c.m.)*			
1.									
2.									
3.									
4.									
5.									
6.									
7.									
8.									
9.									
10.									

Radius of the capillary '*r*' = *c.m.*

Result:

Surface tension of water at temperature _ _ _ _ °C

= dynes/cm = × N/m

Precautions:

1. The water surface and the capillary must be clean since the surface tension is affected by contamination.
2. Capillary tube must be vertical.
3. Capillary tube should be clean and dry.
4. Capillary tube should be of uniform bore.
5. Since the capillary may be conical in shape, so it would be better to break the capillary at the site of the meniscus and find the diameter at that point. However, this is not permitted in our laboratory.
6. The diameter of the tube must be measured very accurately in two perpendicular directions at the point upto which liquid rises.

Theoretical error:

$$T = r\left(h+\frac{r}{3}\right)\frac{g}{2} = \frac{gD}{4}\left(h+\frac{D}{6}\right)$$

Where D is the diameter of capillary tube.
Taking log and differentiating,

$$\frac{\delta T}{T} = \frac{\delta D}{D} + \frac{\delta h}{\left(h+\frac{D}{6}\right)} + \frac{\delta\left(\frac{D}{6}\right)}{\left(h+\frac{D}{6}\right)}$$

Maximum permissible error = %

5.10 OBJECT

To determine the surface tension of a liquid (water) by Jaeger's method.

Apparatus used: Jaeger's apparatus, a glass tube of about 5 mm diameter, a microscope, a scale, beaker, thermometer.

Formula used: The surface tension T of the liquid is given by the formula

$$T = \frac{rg}{2}(H\rho_1 - h\rho), \text{ Newton/meter}$$

where r = radius of the orifice of the capillary tube
g = acceleration due to gravity
H = maximum reading of the manometer just before the air bubble breaks away.
ρ_1 = density of the liquid in the manometer
h = depth of the tip of the capillary tube below the surface of the experimental liquid.
ρ = density of the experimental liquid.
(For water $\rho = 1.00 \times 10^3$ Kg/m^3).

Description of apparatus: The apparatus consists of a Woulf's bottle W fitted in one mouth with a bottle B containing water through a stop-cock K. The other mouth is joined to a manometer M and vertical tube BC as shown in Fig. 5.13. The end C of the tube is drawn into a fine capillary with its cut smooth. For practical purpose, a separate tube C drawn out into a fine jet is connected to the manometer by means of a short piece of Indian rubber J. The end C is kept in the experimental liquid at a known depth of a few cms. The liquid contained in the manometer is of low density in order to keep the difference in the level of liquid in the two limbs of M quite large for a given pressure difference.

Theory: Let the capillary tube be dipped in the liquid, the latter will rise in it and the shape of the meniscus will be approximately spherical. If air is forced into the glass tube by dropping water from the funnel (or burette) into the Woulf's bottle, the surface of the liquid in the capillary tube is pressed downwards and as the pressure of air inside the tube increases the liquid surface goes on sinking lower and lower until finally a hemispherical bubble of radius r equal to that of the orifice of the capillary tube protrudes into the liquid below, the pressure inside the bubble being greater than that outside by

$$p = \frac{2T}{r} \quad ...(1)$$

where T is the surface tension of the liquid.

Suppose that the bubble is formed at the end of a narrow tube of radius r at a depth h_1 in the liquid and that the bubble breaks away when its radius is equal to the radius of the tube. The pressure outside the bubble is $P + h_1\rho_1 g$, where P is the atmospheric pressure and g is the acceleration due to gravity. If the pressure inside the bubble is measured by an open-tube manometer containing a liquid of density ρ_2 then the inside pressure is given by $P + h_2\rho_2 g$, where h_2 represents the difference in heights of the liquid in the two arms of the manometer at the moment the bubble breaks away. Therefore the excess pressure p inside the bubble is $p = (P + h_2\rho_2 g) - (P + h_1\rho_1 g) = g\,(h_2\rho_2 - h_1\rho_1)$.

Hence the surface tension T of the liquid is

$$T = \frac{gr}{2}(h_2\rho_2 - h_1\rho_1) \quad ...(2)$$

In deriving the expression (2) we have assumed that the bubble breaks away when its radius becomes equal to that of the tube. This assumption is not quite correct and so the above expression for T is inaccurate. The correct expression for T is given by

$$T = \frac{1}{2} f(r)\; g\,(h_2\rho_2 - h_1\rho_1) \quad ...(3)$$

where $f(r)$ is an unknown quantity having the same dimension as r. This is Jaeger's formula and is also the working formula of the present experiment. Here $f(r)$ is first obtained from the given value of the surface tension of water at room temperature using Eq. (3). With this value of $f(r)$, the surface tension of water at other temperatures are determined.

If the quantities on the right-hand side of Eq. (3) are expressed in the CGS System of Units, T is obtained in dyne/cm.

Procedure

1. Take a clean glass tube having one end into a fine jet. Hang this tube vertically inside the experimental liquid (water) from the tube *B* with the help of Indian rubber joint *J*.
2. Before proceeding with experiment the apparatus is made air tight.
3. Now the liquid in the tube *BC* stands at a certain level higher than that in the outer vessel. The stop cock *K* is opened slightly so that water slowly falls into the bottle *W* and forces an equal volume of air into the tube *BC*. The liquid in the tube *BC*, therefore, slowly goes down and forms a bubble at the end *C*. A difference of pressure between inside and outside of apparatus is at once set up which is shown by the manometer.
4. The radius of bubble gradually decreases as the inside pressure increases till it reaches the minimum value. At this stage the shape of the bubble is nearly that of a hemisphere of the radius r equal to the radius of the opening at *C*.
5. The difference in the level of the liquid in the two limbs of the manometer is now maximum, say, H and is noted. The bubble now becomes unstable since a small increase in the radius decreases the internal pressure necessary to produce equilibrium. As the external pressure is constant, there can be no more equilibrium state and hence the bubble breaks away.

6. With the help of scale, measure the depth of the orifice below the level of water.
7. Repeat the whole process, fixing the orifice C at different depths below the surface of water in the beaker.
8. The radius of end C is found by a microscope.

Observations: (I) Table for the measurement of H and h

S. No.	*Depth of the orifice below the level of liquid h meter*	*Manometer one limb x meter*	*reading other limb y meter*	*H = (x ~ y) meter*	*Mean H meter*

(II) Temperature of water = ... C°

(III) Table for the measurement of diameter of orifice.

$$\text{Least count of the microscope} = \frac{\text{Value of one div. on main scale in cm}}{\text{Total no. of divisions on vernier scale}} = \text{...... cm}$$

S. No.	*Microscope reading along any direction (↑)*						*Difference (b ~ a) X – cms*	*Microscope reading along a perpendicular direction (→)*						*Difference (d – c) Y – cm*	*Mean diameter (x + y)/2 = D*	*Mean radius r = D/2 cm*
	Left edge of Capillary			*Right edge of Capillary*				*Lower edge of Capillary*			*Upper edge of Capillary*					
	M.S.	*V.S.*	*Total a cm*	*M.S.*	*V.S.*	*Total b cm*		*M.S.*	*V.S.*	*Total c cm*	*M.S.*	*V.S.*	*Total d cm*			
1																
2																
3																
4																

Mean r = ... cm = ... meter

Calculations: $T = \frac{rg}{2}(H - h)$ $\rho_1 = \rho = 100 \times 10^3$ kg/m^3, for water being in beaker and manometer

= ...... Newton/meter

Result: The surface tension of water at ...... °C = ... dyne/cm.

Standard value ...... °C = ...

Error = ...

Graphical method: Draw a graph as shown in Fig. 5.12.

The expression $T = \frac{rg}{2}(H - h)$ can be put in the form $H = h + \frac{2T}{rg}$ which represents a straight line $y = mx + C$ and hence on plotting H on y–axis and h on x-axis a straight line will be obtained, the intersection of which with y-axis will be $\frac{2T}{rg}$.

From graph $\frac{2T}{rg} = OB$

or $T = \frac{rg}{2} \times OB$

$= ......$ Newton/meter

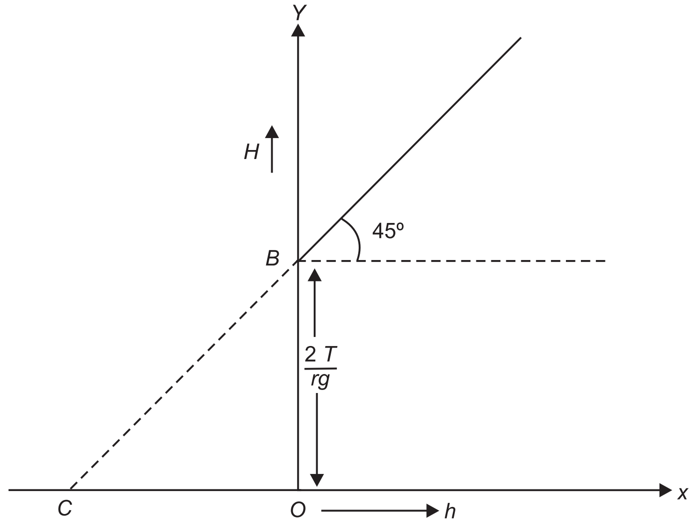


Fig. 5.12

Sources of error and precautions

1. There should be no leakage in the apparatus. It is distinctly advantageous to have the apparatus in one piece avoiding the use of rubber joints.
2. The manometer should contain xylol so that the height H may be large, the density of xylol is less than that of water.
3. To damp the oscillations of the liquid in the manometer, its open end may be drawn out to a capillary tube.
4. Diameter of the capillary tube must be measured only at the orifice dipping in the liquid.
5. The orifice of the capillary tube should be circular and very small, about 0.3 mm in diameter, so that the maximum pressure in a bubble may occur when it is hemispherical.
6. While measuring the diameter of the orifice of the capillary tube, several observations of mutually perpendicular diameters should be taken. This reduces the error due to ellipticity of the cross-section to minimum.

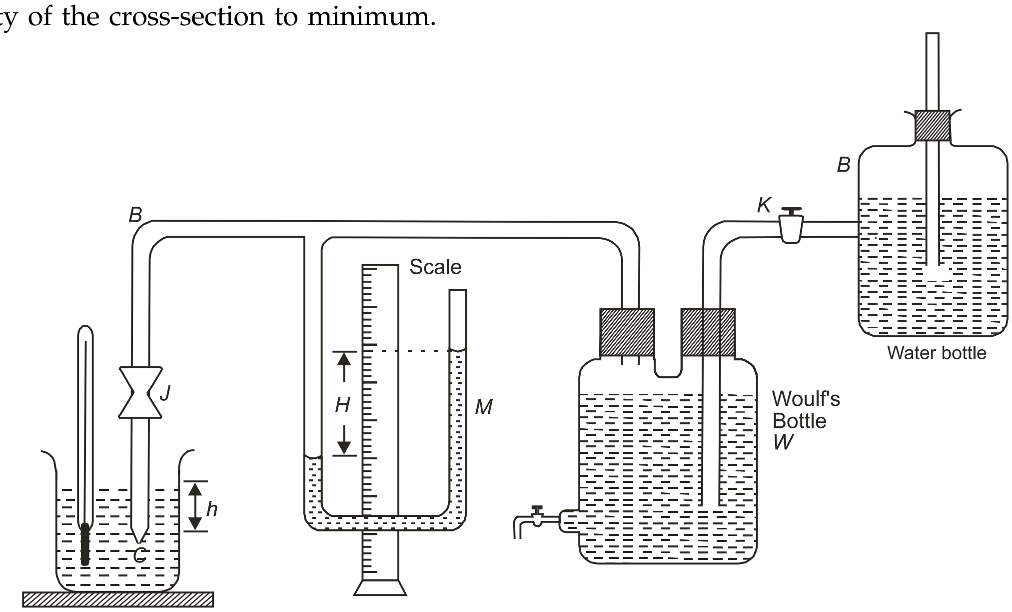


Fig. 5.13

7. In the case of a micrometer microscope error due to backlash should be avoided while taking observations for diameter.
8. The capillary tube should be clean. All traces of grease must be carefully removed as they are fatal to surface tension experiments.
9. As the surface tension of a liquid depends upon its temperature, the temperature of the experimental liquid should be recorded.
10. The bubbles should be formed singly and slowly, say at the rate of one in about ten seconds.

5.11 VIVA-VOCE

Q. 1. What do you understand by the phenomenon of surface tension?

Ans. The free surface of a liquid behaves as if it were in a state of tension having a natural tendency to contract like a stretched rubber membrane. This tension or pull in the surface of a liquid is called surface tension.

Q. 2. How do you define surface tension?

Ans. When a straight line of length unity is drawn on the liquid surface, the portions of the surface on both sides of the line tend to draw away from each other with a force which is perpendicular to the line and tangential to the liquid surface. This force is called surface tension.

Q. 3. What are the units and dimensions of surface tension?

Ans. The C.G.S units of surface tension is dyne/cm and its S.I. unit is Newton/m.

Q. 4. What do you mean by 'cohesive' and 'adhesive forces'?

Ans. The force of attraction between the molecules of the same substance is known as cohesive force while the force of attraction between the molecules of different substances is known as adhesive force.

Q. 5. What do you mean by surface energy of a liquid?

Ans. Whenever a liquid surface is enlarged isothermally, certain amount of work is done which is stored in the form of potential energy of the surface molecule. The excess of potential energy per unit area of the molecules in the surface is called surface energy.

Q. 6. What do you mean by angle of contact?

Ans. The angle between the tangent of the liquid surface at the point of contact and solid surface inside the liquid is known as angle of contact between the solid and the liquid.

Q. 7. What is the value of angle of contact of (*i*) water and glass, and (*ii*) mercury and glass?

Ans. (*i*) 8° and (*ii*) 135°

Q. 8. What is the effect of temperature on surface tension?

Ans. It decreases with rise in temperature and becomes zero at the boiling point of the liquid.

Q. 9. What is the effect of impurities on surface tension?

Ans. The soluble impurities generally increases the surface tension of a liquid while contamination of a liquid surface by impurities (dust, grease etc) decreases the surface tension.

Q. 10. Do you think that surface tension depends only upon the nature of the liquid?

Ans. No, this also depends upon the nature of surrounding medium.

Q. 11. Why does water rise in the capillary tube?

Ans. The vertical component of the surface tension acting all along the circle of contact with the tube ($2\pi rT\cos\theta$) pulls the water up.

Q. 12. How far does the water rise?

Ans. The water in the capillary tube rise only upto a height till the vertical component of surface tension is balanced by weight of water column in capillary.

Q. 13. On what factors the rise of water depend?

Ans. It depends upon: (*i*) surface tension of water (*ii*) Angle of contact and (*iii*) Radius of the tube.

Q. 14. Is it that the liquid always rises in a capillary?

Ans. No, the level of mercury in a capillary tube will be depressed.

Q. 15. How will the rise be affected by (*a*) using capillary of non-uniform bore, (*b*) changing radius of the capillary (*c*) change in shape of the bore.

Ans. (*a*) No effect since the rise of liquid in the capillary depends upon the radius of the tube at the position of the meniscus. Theoretically there is no difficulty if we use a conical capillary.

(*b*) The capillary exent is inversely proportional to the radius, hence change in radius will affect the height of liquid column.

(*c*) No effect.

Q. 16. In the experiment of capillary rise where do you measure the radius of the tube and why?

Ans. The radius is measured at the position of the meniscus because the force of surface tension balancing the water column corresponds to this radius. If the capillary is of uniform bore, then r can be measured at the end.

Q. 17. Will water rise to the same height if we use capillaries of the same radius but of different materials?

Ans. Since surface tension of a liquid depends upon the surface in contact, therefore, in capillaries of different materials, the rise of water will be different.

Q. 18. In your formula for calculating the value of surface tension you have added $\frac{r}{3}$ to the observed height '*h*'. Why have you done so?

Ans. In the experiment '*h*' is measured from free surface of water to the lowest point of the meniscus. To take into account the weight of the liquid above this point. We add this correcting factor $\frac{r}{3}$.

Q. 19. What will be the effect of inclining the tube?

Ans. The vertical height of column will be the same. If l is the length of column. α the angle that capillary makes with vertical and h the vertical height, $l\cos\alpha = h$.

Q. 20. What happens when a tube of insufficient height is dipped in a liquid? Does the liquid over-flow, How is the equilibrium established in this case?

Ans. No, the liquid will not over-flow. The liquid rises to the top of the tube, slightly spreads itself there and adjusts its radius of curvature to a new value R' such that $R'l = Rh$ where l is the length of the tube of insufficient height, R its radius and h the height of column as demanded by formula. In this case $R' > R$ and meniscus becomes less concave.

Q. 21. How will the rise of liquid in a capillary tube be affected if its diameter is halved?

Ans. The height to which the liquid rises, will be doubled.

Q. 22. Why do you measure the diameter in two mutually perpendicular directions?

Ans. This minimises the error due to ellipticity of the bore of the capillary.

Q. 23. Why should the capillary be kept vertical while measuring capillary ascent?

Ans. If the tube is not kept vertical the meniscus will become elliptical and the present formula will not hold.

Q. 24. Should the top of the capillary tube be open or closed?

Ans. It should be open.

Q. 25. Is there any harm if the top of the capillary is closed?

Ans. The rise of water will not be completed as the air above water presses it.

Q. 26. What may be the possible reason if quite low value of surface tension is obtained by this method?

Ans. It may be due to contamination of the capillary with grease or oil.

Q. 27. How can you test that the tube and the water are not contaminated?

Ans. The lower end of the capillary is dipped to a sufficient depth inside water. The water rises in the capillary. The capillary is then raised up. If water falls rapidly as the tube is raised, this shows that water and capillary tube are not contaminated.

Q. 28. For what type of liquids is this method suitable?

Ans. Which wet the walls of the tube and for which the angle of contact is zero.

Q. 29. Can you study the variation of surface tension with temperature by this method?

Ans. No, as the temperature of liquid at the meniscus can not be determined with the required accuracy.

Q. 30. What type of capillary tube will you choose for this experiment?

Ans. A capillary of small circular bore should be chosen.

Q. 31. Water wets glass but mercury does not why?

Ans. The cohesive forces between the water molecules are less than the adhesive forces between the molecules of water and the glass. On the other hand, the cohesive forces between the mercury molecules are larger than the adhesive forces existing between the molecules of the mercury and the glass. That is why water wets glass but mercury does not.

Q. 32. What is the principle underlying Jaeger's method?

Ans. This method is based on the fact that there is always an excess of pressure inside a spherical air bubble formed in a liquid. The excess of pressure $P = \frac{2T}{r}$ where T is surface tension of liquid and r is the radius of the bubble.

Q. 33. Does the excess of pressure depend upon the depth of the orifice, below the surface of experimental liquid?

Ans. No, it depends upon the surface tension of the liquid and radius of bubble.

Q. 34. After breaking, does the size of the bubble remain same as it is near the surface of the liquid.

Ans. No, the size goes on increasing because hydrostatic pressure goes on decreasing as bubble comes near the surface.

Q. 35. Do you think that the radius of the bubble, as considered here, is always equal to the radius of the orifice?

Ans. No, this is only true when the radius of the orifice of capillary tube is very small.

Q. 36. What should be the rate of formation of bubbles?

Ans. The rate of formation of bubble should be very slow, i.e., one bubble per ten second.

Q. 37. What type of capillary will you choose for this experiment?

Ans. We should use a capillary of small circular bore.

Q. 38. What is the harm if the tube is of larger radius?

Ans. When the tube is of large radius the excess of pressure inside the bubble will be small and cannot be measured with same degree of accuracy.

Q. 39. Will there be any change in the manometer reading if the depth of the capillary tube below the surface of liquid is increased?

Ans. Yes, it will increase.

Q. 40. At what position of the levels should the reading be noted?

Ans. The reading should be taken when the manometer shows the greatest difference of pressure because at this moment the bubble breaks.

Q. 41. Can you study the variation of surface tension with temperature with this apparatus?

Ans. Yes, the experimental liquid is first heated and filled in bottle. Now the manometer readings are taken at different temperatures.

Q. 42. Mention some phenomena based on 'surface tension'.

Ans. (*i*) Calming of waves of oil, (*ii*) floating of thin iron needle on water, (*iii*) gyration of camphor on water, etc.

Q. 43. Why is Xylol used as a manometric liquid in preference to water?

Ans. The density of Xylol is lower than that of water. This gives a larger difference of levels in the two limbs of the manometer tube.

Q. 44. Why is the open end of the manometer drawn out into a capillary?

Ans. To minimise the oscillations of the manometric liquid due to surface tension.

Q. 45. At what temperature surface tension is zero.

Ans. At critical temperature.

EXERCISE

Q. 1. Why does liquid surface behave like a stretched rubber membrane?
Q. 2. Do you know any other form of the definition of surface tension?
Q. 3. How do you say that work is done in enlarging the surface area of a film?
Q. 4. Distinguish between surface tension and surface energy?
Q. 5. Mercury does not stick to the finger but water does, explain it?
Q. 6. What is the shape of a liquid surface when it is in contact with a solid?
Q. 7. When will the meniscus be convex or concave and why?
Q. 8. Is there any practical use of the property of surface tension?
Q. 9. Can't you use a wider glass tube instead of a capillary tube? if not why?
Q. 10. How do you explain the shape of meniscus of a liquid in the capillary tube?
Q. 11. Why is it difficult to introduce mercury in a capillary tube?
Q. 12. How do you measure the radius at the meniscus?
Q. 13. How will you clean the capillary tube?
Q. 14. Why is the pressure inside an air-bubble greater than that outside it.
Q. 15. Why does the apparatus be in one piece.

Q. 16. Why does the liquid in the manometer rise and fall during the formation and breaking away of the bubble?

Q. 17. How can you damp the oscillations of liquid in the manometer?

Q. 18. How will you make sure that the bubbles are being formed at the same depth?

Q. 19. Why should the bubbles be formed singly and slowly? How will you accomplish this adjustment?

Q. 20. In measuring the diameter of the tube, why do you take readings along two mutually perpendicular diameters?

Q. 21. What are the advantages of this method?

Q. 22. How can variation of surface tension with temperature be studied with the help of this apparatus?

3

Elasticity

3.1 ELASTICITY

If an external force is applied on a body such that it does not shift the body as a whole, it produces a change in the size or shape or both of the body. In such a case the body is said to be deformed and the applied external force is called the deforming force. This force disturbs the equilibrium of the molecules of the solid bringing into picture the internal forces which resist a change in shape or size of the body. When the deforming force is removed, these internal forces tend to bring the body to its original state. This property of a body by virtue of which it resists and recovers from a change of shape or size or both on removal of deforming force is called its elasticity. If the body is able to regain completely its original shape and size after removal of deforming force it is called perfectly elastic while, if it completely retains its modified size and shape, it is said to be perfectly plastic.

It need hardly be pointed out that there exists no such perfectly elastic or perfectly plastic bodies in nature. The nearest approach to the former is a quartz fibre and to the latter, ordinary putty. All other bodies lie between these two extremes.

3.2 LOAD

Any combination of external forces acting on a body (*e.g.*, its own weight, along with the forces connected with it, like centrifugal force, force of friction etc.) whose net effect is to deform the body, *i.e.,* to change its form or dimensions, is referred to as a load.

3.3 STRESS

A body in equilibrium under the influence of its internal forces is, as we know, in its natural state. But when external or deforming forces are applied to it, there is a relative displacement of its particles ad this gives rise to internal forces of reaction tending to oppose and balance the deforming forces, until the elastic limit is reached and the body gets permanently deformed. The body is then said to be stressed or under stress.

If this opposing or recovering force be uniform, *i.e.*, proportional to area, it is clearly a distributed force like fluid pressure and is measured in the same manner, as force per unit area, and termed stress.

If $\vec{F}$ be the deforming force applied uniformly over an area A, we have stress = $\frac{F}{A}$. If the deforming force be inclined to the surface, its components perpendicular and along the surface are respectively called normal and tangential (or shearing) stress. The stress is, however, always normal in the case of a change of length or volume and tangential in the case of a change of shape of a body.

Its dimensional formula is $ML^{-1} T^{-2}$ and its units in M.K.S. and C.G.S. systems are respectively newton/m^2 and dyne/cm^2.

3.4 STRAIN

When the body is deformed on application of external force, it is said to be strained. The deformation of the body is quantitatively measured in terms of strain which is defined as the change in some dimension of the body per unit that dimension. A body can be deformed in three ways *viz.* (*i*) a change in length producing longitudinal strain, (*ii*) a change in volume producing volume strain and, (*iii*) a change in shape without change in volume (*i.e.*, shearing) producing shearing strain. Thus change in length per unit length is called longitudinal strain, change in volume per unit volume is called volume strain, and change in an angle of body is called shearing strain being a ratio the strain is a dimensionless quantity and has no unit.

3.5 HOOKE'S LAW

This law states that when the deforming force is not very large and strain is below a certain upper limit, stress is proportional to strain *i.e.*,

$$\text{stress} \propto \text{strain}$$

$$\frac{\text{stress}}{\text{strain}} = \text{constant.}$$

This ratio of stress and strain, which is constant, is known as the modulus of elasticity and depends upon the material of the body. The limit upto which Hooke's law holds is called the limit of elasticity. Thus Hooke's law may be stated as within elastic limit, stress is proportional to strain.

3.6 ELASTIC LIMIT

In the case of a solid, if the stress be gradually increased, the strain too increases with it in accordance with Hooke's law until a point is reached at which the linear relationship between the two just ceases and beyond which the strain increases much more rapidly than is warranted by the law. This value of the stress for which Hooke's law just ceases to be obeyed is called the elastic limit of the material of the body for the type of stress in question.

The body thus recovers its original state on removal of the stress within this limit but fails to do so when this limit is exceeded, acquiring a permanent residual strain or a permanent set.

3.7 TYPES OF ELASTICITY

Since in a body there can be three types of strain *viz.* , longitudinal strain, volume strain and the shearing strain, correspondingly we have three types of elasticity as described below.

3.8 YOUNG'S MODULUS (OR ELASTICITY OF LENGTH)

When the deforming force is applied to a body in such a manner that its length is changed, longitudinal or linear strain is produced in the body. The internal force of reaction or the restoring force trying to restore its length, restoring force acts along the length of the body and its magnitude per unit cross-sectional area is the normal stress. The ratio of this normal stress and the linear strain is called Young's modulus of elasticity Y.

Thus if a uniform wire of length L and cross-section area A is stretched in length by an amount l by a force F acting along its length, the internal restoring force equals the external force in the equilibrium state, then

$$\text{Longitudinal strain} = \frac{l}{L}$$

and

$$\text{Normal stress} = \frac{F}{A}$$

$$\therefore \qquad Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{\dfrac{F}{A}}{\dfrac{l}{L}}$$

The dimensional formula for Young's modulus is ($ML^{-1}T^{-2}$) and its units in M.K.S. and C.G.S. systems are respectively newton/metre^2 and dyne/cm^2.

3.9 BULK MODULUS (OR ELASTICITY OF VOLUME)

When a force is applied normally over the whole surface of the body, its volume changes while its shape remains unchanged. In equilibrium state, internal restoring force equals the external force. The magnitude of normal force per unit area is the normal stress. This may also be appropriately called pressure. The ratio of normal stress and the volume strain is called bulk modulus of elasticity, K.

Thus, if v is the change in volume produced in the original volume V of the body by application of force F normally on surface area A of the body, we have

$$\text{Volume strain} = \frac{v}{V}$$

$$\text{Normal stress} = \frac{F}{A} = p$$

$$\therefore \qquad \text{Bulk modulus} = -\frac{\text{Pressure}}{\text{Volume strain}}$$

or

$$K = -\frac{P}{v/V}$$

The minus sign has been introduced to give K a positive value. This is because an increase in pressure (p-positive) causes a decrease in volume (V-negative) of the body.

K is also sometimes referred to as incompressibility of the material of the body and, therefore, $\frac{1}{K}$ is called its compressibility.

Since liquids and gases can permanently sustain only a hydrostatic pressure, the only elasticity they possess is Bulk modulus (K).

The dimensional formula for bulk modulus of elasticity is [$ML^{-1} T^{-2}$] and its units in MKS and C.G.S. systems are newton/metre^2 and dyne/cm^2 respectively.

3.10 MODULUS OF RIGIDITY (TORSION MODULUS OR ELASTICITY OF SHAPE)

When under application of an external force the shape of the body changes without change in its volume, the body is said to be sheared. This happens when a tangential force is applied to one of the faces of the solid.

Consider a rectangular solid *ABCD abcd* whose lower face *DC cd* is fixed and a tangential force F is applied to its upper face. *ABba* as shown in figure. The layers parallel to the lower face slip through distances proportional to their distance from the fixed face such that finally face *ABba* shifts to *A′B′b′a′* and solid takes the form *A′B′C Da′b′cd* its volume remaining unchanged. Thus one face of solid remains fixed while the other is shifted laterally. The angle $ADA' = \theta = \left(=\frac{l}{L} \text{ where } AD=L \text{ and } AA'=l\right)$ through which the edge AD which was initially perpendicular to the fixed face is turned, is called the shearing strain or simply the shear. Due to this shearing of the solid, tangential restoring force is developed in the solid which is equal and opposite to the external force. The ratio of tangential stress and shearing strain is called modulus of rigidity, η. Thus

$$\text{Tangential stress} = \frac{F}{A}$$

$$\text{shearing strain } \theta = \frac{AA'}{AD} = \frac{l}{L}$$

$$\text{Modulus of rigidity} = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$\eta = \frac{\frac{F}{A}}{\theta}$$

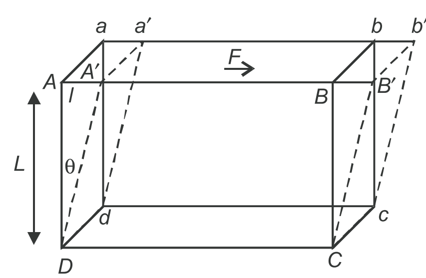


Fig. 3.1

where A is the area of the upper face of the solid.

The dimensional formula for modulus of rigidity is $ML^{-1}T^{-2}$ and its MKS and CGS units are newton/m^2 and dyne/cm^2 respectively.

3.11 AXIAL MODULUS

This is defined as longitudinal stress required to produce unit linear strain, unaccompanied by any lateral strain and is denoted by the Greek letter Z. It is thus similar to Young's modulus, with the all-important difference that here the lateral strain produced (in the form of lateral contraction) is offset by applying two suitable stresses in directions perpendicular to that of the linear stress. So that the total stress is Young's modulus plus these two perpendicular stresses.

3.12 POISSON'S RATIO

When we apply a force on a wire to increase its length, it is found that its size change not only along the length but also in a direction perpendicular to it. If the force produces an extension in its own direction, usually a contraction occurs in the lateral or perpendicular direction and vice-versa. The change in lateral dimension per unit lateral dimension is called lateral strain. Then, within elastic limit, the lateral strain (though opposite in sign) is proportional to the longitudinal strain *i.e.*, the ratio of lateral strain to the longitudinal strain within the limit of elasticity is a constant for the material of a body and is called the Poisson's ratio. It is usually denoted by σ.

Consider a wire of length L and diameter D. Under application of an external longitudinal force F, let l be the increase in the length and d the decrease in the diameter, then

$$\text{Longitudinal strain } \alpha = \frac{l}{L}$$

$$\text{Lateral strain } \beta = \frac{d}{D}$$

$$\therefore \quad \text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\sigma = \frac{\beta}{\alpha} = \frac{\frac{d}{D}}{\frac{l}{L}}$$

Being ratio of two strains, Poisson's ratio has no units and dimensions.

3.13 RELATION BETWEEN ELASTIC CONSTANT

In the above discussion we have defined four elastic constants, *viz.* Y, K, η and σ. Out of these only two are independent while other two can be expressed in terms of the two independent constants. Hence if any two of them are determined, the other two can be calculated. The following are the inter-relations between the four elastic constants.

(*i*) $Y = 3K(1 - 2\sigma)$

(*ii*) $Y = 2\eta(1 + \sigma)$

(*iii*) $Y = \dfrac{9K\eta}{3K + \eta}$

(*iv*) $\sigma = \dfrac{3K - 2\eta}{6K + 2\eta}$

3.14 LIMITING VALUES OF POISSON'S RATIO (σ)

From the relation $3k(1 - 2\sigma) = 2\eta(1 + \sigma)$, where, as we know, K and η are both positive quantities it follows therefore, that

(*i*) If σ be a positive quantity, the expression on the right hand side in the relation above will be positive. The expression on the left hand side too must therefore be positive. This is, obviously, possible when $2\sigma < 1$ or $\sigma < \frac{1}{2}$ or 0.5; and

(*ii*) If σ be a negative quantity, the left hand expression in the above relation will be positive and hence the expression on the right hand side too must be positive, and this can be so only if σ be not less than –1.

Thus, theoretically, the limiting values of σ are –1 and 0.5, though in actual practice it lies between 0.2 and 0.4 for most of the materials.

3.15 TWISTING COUPLE ON A CYLINDER

(*i*) *Case of a solid cylinder or wire:* Let a solid cylinder (or wire) of length L and radius R be fixed at its upper end and let a couple be applied to its lower end in a plane perpendicular to its length (with its axis coinciding with that of the cylinder) such that it is twisted through an angle θ.

This will naturally bring into play a resisting couple tending to oppose the twisting couple applied, the two balancing each other in the position of equilibrium.

To obtain the value of this couple, let us imagine the cylinder to consist of a large number of hollow, coaxial cylinder, one inside the other and consider one such cylinder of radius x and thickness dx, As will be readily seen, each radius of the base of the cylinder will turn through the same angle θ but the displacement (BB') will be the maximum at the rim, progressively decreasing to zero at the centre (O) indicating that the stress is not uniform all over.

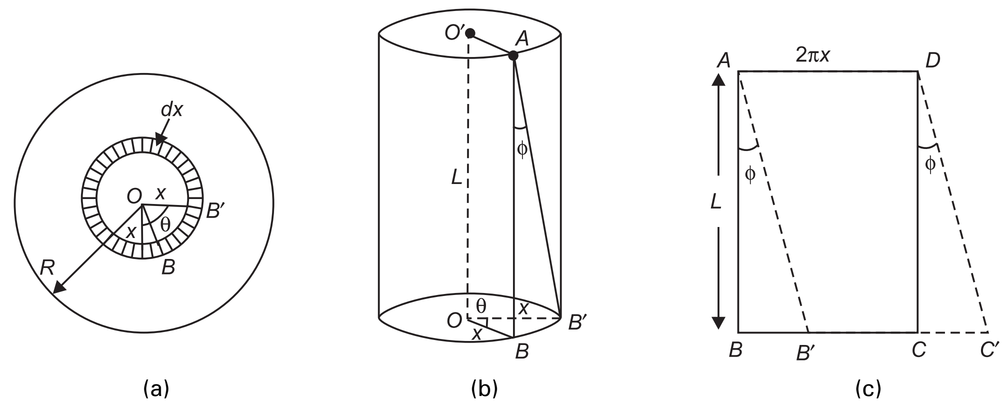


Fig. 3.2

Thus, a straight line AB, initially, parallel to the axis OO' of the cylinder will take up the position AB' or the angle of shear (or shear) = $\angle BAB' = \phi$. This may be easily visualised if we imagine the hollow cylinder to be cut along *AB* and spread out when it will initially have a rectangular shape *ABCD*, and will acquire the shape of a parallelogram *AB′C′D* after it has been twisted, so that angle of shear = $BAB' = \phi$.

Now, $BB' = x\theta = L\phi$, where, the shear $\phi = \dfrac{x\theta}{L}$ and will obviously have the maximum value when $x = R$, *i.e.*, at the outermost part of the cylinder and the least at the innermost.

If η be the coefficient of rigidity of the material of the cylinder, we have

$$\eta = \frac{\text{shearing stress}}{\text{shear}} \text{ or shearing stress} = \eta \times \text{shear}$$

or $$\text{shearing stress} = \eta\phi = \frac{\eta x\theta}{L}$$

∴ Shearing force on face area of the hollow cylinder $= \left(\dfrac{\eta x\theta}{L}\right) \times$ face area of the cylinder

$$= \left(\frac{\eta x\theta}{L}\right) \times 2\pi x dx = \left(\frac{2\pi\eta\theta}{L}\right) x^2\, dx$$

And the moment of the force about the axis *OO′* of the cylinder

$$= \left(\frac{2\pi\eta\theta}{L}\right) x^2\, dx \cdot x = \left(\frac{2\pi\eta\theta}{L}\right) x^3\, dx$$

∴ twisting couple on the whole cylinder

$$= \int_0^R \frac{2\pi\eta\theta}{L} x^3\, dx = \frac{\pi\eta R^4}{2L}\theta$$

or, twisting couple per unit twist of the cylinder or wire, also called torsional rigidity of its material, is given by

$$C = \frac{\pi\eta R^4}{2L}$$

(*ii*) *Case of a hollow cylinder*: If the cylinder be a hollow one, of inner and outer radii R_1 and R_2 respectively we have

$$\text{twisting couple on the cylinder} = \int_{R_1}^{R_2} \frac{2\pi\eta\theta}{L} x^3\, dx = \frac{\pi\eta}{2L}\left(R_2^4 - R_1^4\right)\theta$$

∴ twisting couple per unit twist, say $C' = \dfrac{\pi\eta}{2L}(R_2^4 - R_1^4)$

Now, if we consider two cylinders of the same material, of density ρ, and of the same mass *M* and length *L*, but one solid, of radius *R* and the other hollow of inner and outer radii R_1 and R_2 respectively.

We have $$\frac{C'}{C} = \frac{R_2^4 - R_1^4}{R^4} = \frac{(R_2^2 + R_1^2)\left(R_2^2 - R_1^2\right)}{R^4}$$

Since $M = \pi(R_2^2 - R_1^2)\, L\rho = \pi R^2 L\rho$, we have $(R_2^2 - R_1^2) = R^2$

or $$\frac{C'}{C} = \frac{(R_2^2 + R_1^2)R^2}{R^4} = \frac{(R_2^2 + R_1^2)}{R^2}$$

Again, because $R_2^2 - R_1^2 = R^2$, we have $R_2^2 = R^2 + R_1^2$

And, therefore, $R_2^2 + R_1^2 = R^2 + R_1^2 + R_1^2 = R^2 + 2R_1^2$, *i.e.*,

$$(R_2^2 + R_1^2) > R^2.$$

Clearly, therefore, $\frac{C'}{C} > 1$ or $C' > C$

or, the twisting couple per unit twist is greater for a hollow cylinder than for a solid one of the same material, mass and length.

This explains at once the use of hollow shafts, in preference to solid ones, for transmitting large torque in a rotating machinery.

3.16 OBJECT

To determine the value of modulus of rigidity of the material of a wire by statical method using vertical pattern apparatus (Barton's apparatus).

Apparatus: Barton's vertical pattern torsion apparatus, a wire, screw gauge, a vernier callipers, meter scale, set of weights, thread and a meter scale.

Formula Used: The modulus of rigidity η is given by

$$\eta = \frac{2\,mg\,Dl}{\pi\theta\, r^4}$$

where m = load suspended from each pan
g = acceleration due to gravity
D = Diameter of the heavy cylinder
r = radius of the experimental wire
l = length of the wire
θ = angle of twist in degree

Description of apparatus: The vertical pattern of torsion apparatus (Barton's apparatus) shown in figure is used for specimen available in the form of a long thin rod, whose upper end is fixed securely to a heavy metallic frame and lower end is fixed to a heavy metal cylinder *C*. This heavy cylinder keeps the wire vertical. Flexible cord attached to two diametrically opposite pegs on the cylinder leave it tangentially diametrically opposite points after half a turn. These cords pass over two frictionless pulleys P_1 and P_2, fixed in the heavy frame, and at their free end carry a pan each of equal weight. When equal loads are placed on the pans, couple acts on the cylinder which produces a twist in the rod. The twists are measured by double ended pointer which move over the concentric circular scales graduated in degrees. The three levelling screws are provided at the base of the metallic frame supporting the rod, to make it vertical. In this case the centres of the scale fall on the axis of the rod.

Theory: The modulus of rigidity (η) is defined as the ratio of shearing stress to shearing strain. The shearing stress is the tangential force *F* divided by the area *A* on which it is applied and the shearing strain is the angle of shear ϕ, therefore:

$$\eta = \frac{\frac{F}{A}}{\phi}$$

or $$F = \eta A \phi \qquad \text{...(1)}$$

The twisting of a wire can be related to shearing as follows. Consider a solid cylinderical wire (length l and radius r) as consisting of a collection of thin hollow cylindrical elements. One such element is shown shaded in Fig. 3.5 (*b*). A line PQ of length l is drawn on the surface of the element parallel to its axis OO'. If the wire is clamped at the top and the bottom is twisted through the same angle and the line PQ would become PQ'. The same picture is shown in cross-section in Fig. 3.5(*b*). If we consider this element only, cut it along the line PQ and unroll it, the picture would be seen in Fig. 3.4. This picture is similar to Fig. 3.2(c), with an area A, being given by $2\pi x dx$ and the shearing angle being $QPQ' = \phi$

From Fig. 3.4(*b*) (if ϕ is small), the arc QQ' is equal to $(l \times \phi)$ and from Fig. 3.5 (*b*), the arc QQ' is equal $(x \times \theta)$. Therefore $l\phi = x\theta$

$$\phi = \frac{x\theta}{l} \qquad \text{...(2)}$$

From Fig. 3.5, we see that the area (A) on which the shearing tangential force is applied to the element $A = 2\pi x dx$.

Substituting for A and ϕ in Eq. (1), we obtain

$$F = \eta\ 2\pi x dx\ \frac{x\theta}{l}$$

or $$F = 2\pi\eta \frac{x^2\theta}{l} dx$$

The moment of this force about the axis of the element (OO') is

$$F \times x = 2\pi\eta \frac{x^3\theta}{l} dx$$

We can calculate the total moment required to twist the entire solid cylinder (wire), by integrating over all the elements. The moment, or couple, C is given by

$$C = \int_0^r 2\pi\eta \frac{\theta}{l} x^3\, dx$$

or $$C = \left[2\pi\eta \frac{\theta}{l} \frac{r^4}{4}\right]$$

or $$C = \frac{\pi\eta\theta r^4}{2l} \qquad \text{...(3)}$$

In this experiment, the twisting or shearing moment is provided by the couple applied to the thick heavy cyclinder of diameter D, fixed at the bottom of the wire. This couple is due to the tension in the threads bearing the weights (Fig. 3.3(*b*). Thus $C = mgD$, where mg is the tension in each thread. Therefore,

$$mgD = \frac{\pi\eta\theta r^4}{2l}$$

or $$\eta = \frac{2mgDl}{\pi\theta r^4}$$ where θ is measured in degree

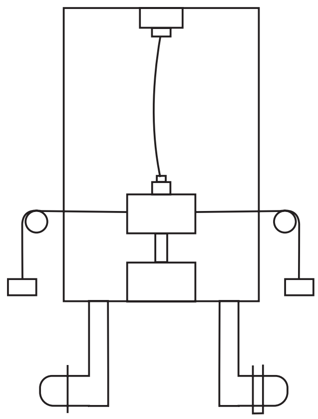


Fig. 3.3(a): Rigid apparatus

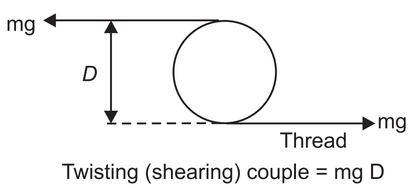


Fig. 3.3(b): Heavy thick solid cylinder

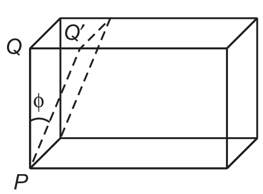


Fig. 3.4: Illustrating shear: shearing angle is $QPQ' = \phi$

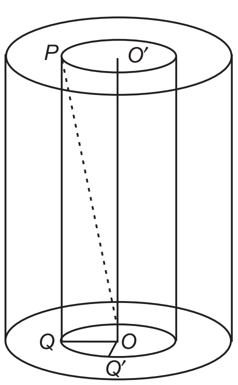


Fig. 3.5(a)

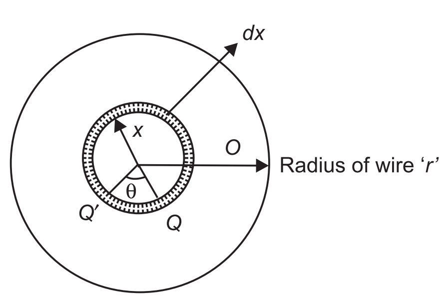


Fig. 3.5(b): Cross section of wire showing element (shaded and the angle of twist θ. $QQ' = x\theta$)

Manipulation

1. Level the base of the Barton's apparatus by the levelling screws at the base using spirit level so that the wire hangs freely vertically and can be twisted without any friction.

2. Wind the thread around the thick cylinder as shown in Fig. 3.3(*b*), pass the two ends over the pulleys and attach the pans to them.
3. Take readings of the pointers on the circular scale with no weights on the pans.
4. Add weights of 10 gm on each pan and take the readings again, repeating this until the total weight on each pan is 50 gms. Take readings again while the weights are reduced to zero.
5. Measure the diameter of the wire using the screw gauge and the diameter of the thick cylinder using a vernier callipers. Take readings of diameters along the entire length in mutually perpendicular directions to correct for any departure from uniform or circular cross-section (some places the area is elliptical hence we measure many places).
6. Measure the length of the wire which is being twisted.
7. Using the vernier callipers measure the diameter of the metallic cylinder.

Observations: (A) Table for the measurement of angle of twist (θ):

S.No.	*Load in each pans (in gm)*	*Pointer readings When load Increasing*		*Decreasing*		*Mean*	*Deflection for 30 gms Load*	*Mean Deflection for 30 gms load*
		Right	*Left*	*Right*	*Left*			
1	0							
2	10							
3	20							
4	30							
5	40							
6	50							

Mean deflection angle for 30 gm load = _______ radian.

(*i*) The wire should not be twisted beyond elastic limits

(*ii*) Table for the measurement of diameter of the given wire.

Least count of screw gauge =

Zero error = ±

S. No.	1	2	3	4	5	6	7	8	9	10	Mean
Diameter in one direction (in cm)											
Diameter perpendicular to above (in cm)											

Diameter of wire corrected for zero error =

Radius r =

(*iii*) Table for the measurement of diameter of the given cylinder

Least count of vernier callipers =

Zero error =

S. No.	1	2	3	4	5	6	7	8	9	10	Mean
Diameter in one direction (in cm)											
Diameter perpendicular to above (in cm)											

Diameter (D) of cylinder corrected for zero error =

(*iv*) Length of the wire l =

Calculations: $$\eta = \frac{2mg\,Dl}{\pi\theta\, r^4}$$

Result: The value of modulus of rigidity, η of the given iron wire as determined from Barton's apparatus is =

Standard value = η for iron = $7.2 - 8.5 \times 10^{10}$ N/m^2

Percentage error = %

$$\text{Theoretical error} = \eta = \frac{2\,mg\,Dl}{\pi\,\theta\,r^4}$$

Taking log, and differentiating, we get

$$\frac{\delta x}{\eta} = \frac{\delta M}{M} + \frac{\delta D}{D} + \frac{\delta l}{l} + \frac{\delta l}{l} + 4\frac{\delta l}{r} + \frac{\delta\theta}{\theta}$$

= __________

Maximum permissible error =

Precautions:

1. First of all base of the instrument should be levelled using sprit level.
2. The wire must be of uniform circular cross-section, free of links, hanging freely and vertically, firmly clamped at the top.
3. Too much weights must not be put on the pans, else the wire may twist beyond elastic limit.
4. The wire should be trained before the readings are taken.
5. The radius of the wire must be taken carefully since its occurs fourth power is occurring in the formula.
6. The pulleys should be frictionless.
7. Load should be increased of decreased gradually and gently.
8. The chord wound round the cylinder should be thin and strong.
9. Before starting experiment ensure that the upper end of the rod is firmly clamped. If it is not so, the rod may slip at this end on application of load.
10. The length of the wire, it measured between the two clamps.

BENDING OF BEAMS

3.17 BEAM

A rod or a bar of circular or rectangular cross-section, with its length very much greater than its thickness (so that there are no shearing stresses over any section of it) is called a beam.

If the beam be fixed only at one end and loaded at the other, it is called a cantilever.

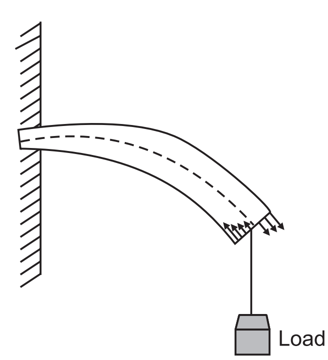


Fig. 3.6

3.18 BENDING OF A BEAM

Suppose we have a beam, of a rectangular cross-section, say, fixed at one end and loaded at the other (within the elastic limit) so as to be bent a little, as shown in figure with its upper surface becoming slightly convex and the lower one concave. All the longitudinal filaments in the upper half of the beam thus get extended or lengthened, and therefore under tension, and all those in the lower half get compressed or shortened and therefore under pressure.

These extensions and compressions increase progressively as we proceed away from the axis on either side so that they are the maximum in the uppermost and the lowermost layers of the beam respectively. There must be a layer between the uppermost and the lowermost layers where the extensions in the upper half change sign to become compressions in the lower half. In this layer or plane, which is perpendicular to the section of the beam containing the axis, the filaments neither get extended nor compressed, *i.e.* retain their original lengths. This layer is therefore called the neutral surface of the beam.

3.19 THEORY OF SIMPLE BENDING

There are the following assumptions:

(*i*) That Hooke's law is valid for both tensile and compressive stresses and that the value of Young's modulus (Y) for the material of the beam remains the same in either case.

(*ii*) That there are no shearing stresses over any section of the beam when it is bent. This is more or less ensured if the length of the beam is sufficiently large compared with its thickness.

(*iii*) That there is no change in cross-section of the beam on bending. The change in the shape of cross-section may result in a change in its area and hence also in its geometrical moment of inertia I_g. Any such change is however always much too small and is, in general, ignored.

(*iv*) That the radius of curvature of the neutral axis of the bent beam is very much greater than its thickness.

(*v*) That the minimum deflection of the beam is small compared with its length.

3.20 BENDING MOMENT

When two equal and opposite couples are applied at the ends of the rod it gets bent. The plane of bending is the same as the plane of the couple. Due to elongation and compression of the filaments above and below the neutral surface, internal restoring forces are developed which constitute a restoring couple. In the position of equilibrium the internal restoring couple is equal and opposite to the external couple producing bending of the beam. Both these couples lie in the plane of bending. The moment of this internal restoring couple is known as bending moment.

Expression for bending moment: Consider the forces acting on a cross-section through *CD* Fig. 3.7 of a bent beam. The external couple is acting on end *B* in the clockwise direction. The filaments above the neutral surface are elongated such that change in length is proportional to their distance from neutral axis. Therefore, the filaments to the right of *CD* and above neutral axis exert a pulling force towards left due to elastic reaction. Similarly, since the filaments below neutral axis are contracted, with change in length proportional to their distance from neutral axis, due to elastic reaction they exert a pushing force towards right as shown in figure.

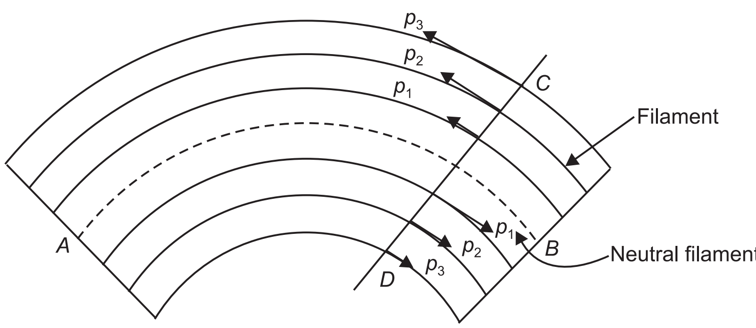


Fig. 3.7

Thus on *CD*, the forces are towards left above neutral axis while they are towards right below it. These forces form a system of anticlockwise couples whose resultant is the internal restoring couple. This couple is equal and opposite to the external couple producing bending in the beam, and keeps the position of beam to the right of *CD* in equilibrium. The moment of this internal restoring couple acting on the cross-section at *CD* is termed as bending moment. Its value is given by

$$G = \frac{YI_g}{R}$$

Where $I_g = \Sigma\delta a.\ z^2$ is called the geometrical moment of inertia of the cross-section of the beam about the neutral filament (This quantity is analogous to the moment of inertia with the difference that mass is replaced by area). For a beam of rectangular cross-section of width *b* and thickness *d*

$$I = \frac{1}{12}bd^3$$

and bending moment $$G = \frac{Ybd^3}{12R}$$

For a beam of circular cross-section of radius *r* its value is

$$I = \frac{1}{4}\pi r^4$$

and Bending moment $$G = \frac{Y\pi r^4}{4R}$$

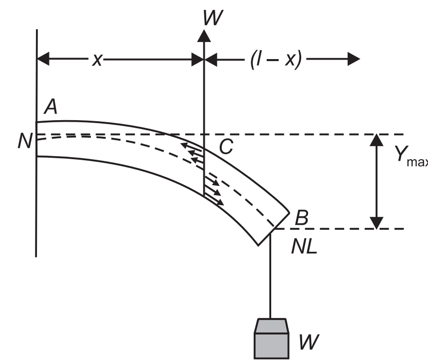
In the position of equilibrium, this bending moment balances the enternal bending couple τ, thus

$$\tau = C = \frac{YI}{R}$$

or $$R = \frac{YI}{\tau}$$

showing that the beam of uniform cross-section is bend into an arc of circle, since *R* is constant for given τ.

3.21 THE CANTILEVER



When a beam of uniform cross-section is clamped horizontally at one end and could be bent by application of a load at or near the free end, the system is called a cantilever.

When the free end of the cantilever is loaded by a weight W (= Mg), the beam bends with curvature changing along its length. The curvature is zero at the fixed end and increases with distance from this end becoming maximum at the free end. This is because of the fact that at distance x from fixed end, for equilibrium of portion CB of the beam the moment of external couple is $W(l - x)$, where l is the length of the cantilever. Thus for equilibrium of portion CB of cantilever, we have

$$G = \frac{YI}{R} = W(l - x)$$

Here it is assumed that the weight of the beam is negligible.

Now the radius of curvature R of the neutral axis at P distant x from fixed end, and having depression y is given by

$$\frac{1}{R} = \frac{d^2y/dx^2}{\left[1 + (dy/dx)^2\right]^{3/2}}$$

where (dy/dx) is the slope of the tangent at print $P(x, y)$. If the depression of the beam is small $\left(\frac{dy}{dx}\right)$ will be very small quantity in comparison to 1 and is, therefore, negligible. Hence

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Thus $$C = YI\frac{d^2y}{dx^2} = W(l - x)$$

or $$\frac{d^2y}{dx^2} = \frac{W}{YI}(l - x)$$

Integrating twice under the condition that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$, we get

$$y = \frac{W}{YI}\left(\frac{lx^2}{2} - \frac{x^3}{\sigma}\right)$$

This gives the depression of the beam at distance x from the fixed end, under the assumption that the weight of the beam itself is negligible.

At the loaded end where $x = l$, the depression is maximum and is given by

$$y_{max} = \delta = \frac{Wl^3}{3YI}$$

If the beam is of rectangular cross-section (breadth b and thickness d), $I = \frac{1}{12} bd^3$, so that

$$\delta = \frac{4Wl^3}{ybd^3}$$

and for beam of circular cross-section of radius r, $I = \frac{\pi r^4}{4}$, so that

$$\delta = \frac{4Wl^3}{3y\pi r^4}$$

3.22 BEAM SUPPORTED AT BOTH THE ENDS AND LOADED IN THE MIDDLE

The arrangement of a beam supported at its both the ends and loaded in the middle is the most convenient method of measurements. A long beam AB of uniform cross-section is supported symmetrically on two knife edges K_1 and K_2 in the same horizontal plane and parallel to each other at a distance l apart. When the beam is loaded at its middle point c by a weight W, this generates two reactions equal to $\frac{W}{2}$ each, acting vertically upwards at the two knife-edges. The beam is bent in the manner as shown in figure. The maximum depression is produced in the middle of the beam where it is loaded.

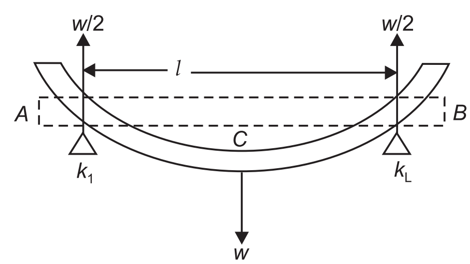


Fig. 3.8

By consideration of symmetry it is clear that the tangent to the beam at C will be horizontal. Hence the beam can be divided into two portions AC and CB by a transverse plane through the middle point of the beam. Each portion can be regarded as a cantilever of length $\frac{l}{2}$, fixed horizontally at point C and carrying a load $\frac{W}{2}$ in the upward direction at the other end (*i.e.*, these are inverted cantilevers). The elevation of the ends A and B above middle point C is equal to the depression of the point C. The depression at the middle point is thus obtained as

$$\delta = \frac{\left(\frac{W}{2}\right) \times \left(\frac{l}{2}\right)^3}{3YI} = \frac{Wl^3}{48YI}$$

$$\delta = \frac{Mgl^3}{48YI} \text{ since } W = Mg$$

Hence for a rectangular beam of breadth b and thickness d

$$\delta = \frac{Mgl^3}{4Ybd^3}$$

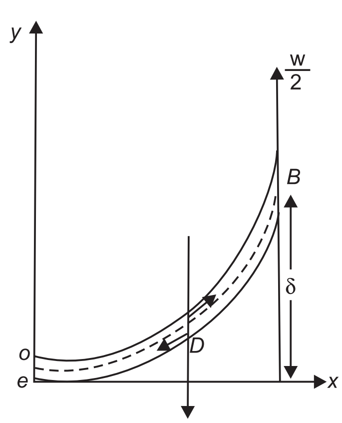


Fig. 3.9

and for a beam of circular cross-section of radius r

$$\delta = \frac{Mgl^3}{12\pi Y r^4}$$

From these expressions, knowing the dimensions of the beam and the depression at the middle point by a known weight, the value of Young's modulus Y, for the material of the beam can be calculated.

Methods of measuring depression: The depression of the beam supported at its ends and loaded in the middle is usually measured by either of the methods *viz.*

(*i*) by micrometer screw or spherometer with some electrical indicating device like a bell, bulb, galvanometer or voltmeter.
(*ii*) by single optical lever method
(*iii*) By double optical lever or Koenig's method

3.23 OBJECT

To determine the Young's modulus of the material of a given beam supported on two knife-edges and loaded at the middle point.

Apparatus used: Two parallel knife edges on which the beam is placed, a hook to suspend weights, a scale attached to the hook, 0.5 kg weights, a cathetometer, a vernier callipers and a meter scale.

Formula used: The Young's modulus (Y) for a beam of rectangular cross-section is given by the relation

$$Y = \frac{Mgl^3}{4bd^3 y} \text{ Newton / meter}^2$$

Where M = load suspended from the beam, g = acceleration due to gravity, l = length of the beam between the two knife edges, b = breadth of the beam, d = thickness of the beam and y = depression of the beam in the middle.

Theory: Let a beam be supported horizontally on two parallel knife edges A and B (Figure 3.8), distance l apart and loaded in the middle C with weight W. The upward reaction at each knife edge being $\frac{W}{2}$ and the middle part of beam being horizontal, it may be taken to be combination of the two inverted cantilevers CA and CB, each of effective length $\frac{l}{2}$ fixed at C and bending upward under a load $\frac{W}{2}$ acting on A and B. Let the elevation of A and B above C or the depression of C below A and B be 'y'.

Consider the section DB of the cantilever CB at a distance x from its fixed end C (Figure 3.9). Bending couple due to load $\frac{W}{2} = \frac{W}{2}\left(\frac{l}{2} - x\right)$. The beam being in equilibrium, this must be just balanced by the bending moment (or the moment of the resistance to bending) $\frac{YI_g}{R}$, where R is the radius of curvature of the section at D and I_g is the geometrical moment of inertia of the cross-section of the beam about an axis passing through the centre of the beam and perpendicular to it. Therefore, we have

$$\frac{YI_g}{R} = YI_g \frac{d^2y}{dx^2} = \frac{W}{2}\left(\frac{l}{2} - x\right) \text{ since } \frac{1}{R} = \frac{d^2y}{dx^2}$$

Which on integration gives $\frac{dy}{dx} = \left[\frac{W}{2YI_g}\right]\left[\frac{lx}{2} - \frac{x^2}{2}\right] + c$

since at $x = 0$, $\frac{dy}{dx} = 0$, we have $c = 0$ and therefore,

$$\frac{dy}{dx} = \left[\frac{W}{2YI_g}\right]\left[\frac{lx}{2} - \frac{x^2}{2}\right]$$

or

$$dy = \left[\frac{W}{2YI_g}\right]\left[\frac{lx}{2} - \frac{x^2}{2}\right]dx$$

Which on further integration between the limits $x = 0$ and $x = \frac{l}{2}$ gives

$$y = \frac{W}{2YI_g}\left[\frac{l^3}{16} - \frac{l^3}{48}\right]$$

$$= \frac{Wl^3}{48YI_g}$$

If the cross-section of the beam be rectangular of breadth 'b' and thickness 'd', we have $I_g = \frac{bd^3}{12}$. Hence above equation can now be written as

$$y = \frac{Wl^3}{4Ybd^3}$$

or

$$y = \frac{Mgl^3}{4Ybd^3}$$

Where M is the mass suspended from the hook. The depression y of the mid-point is noted directly with the help of cathetometer.

Manipulation:

1. Adjust the cathetometer so that
   (*a*) The vertical column that carries the microscope and scale is vertical.
   (*b*) As microscope is moved up and down the column, the axis of the microscope is parallel to some fixed direction in the horizontal plane.

2. Support the experimental beam symmetrically on the knife edges with equal lengths projecting beyond the knife edges.
3. Measure the distance between knife edges with a meter scale. This gives the length l of the beam under flexure.
4. Suspend the hanger with a graduated scale attached to it, on the mid-point of the beam. Focus the microscope horizontal cross-wire of the microscope on a certain division of the scale. Take the reading of the vernier scale of cathetometer (observation table I)
5. Suspend a weight of 1 kg on the hanger. A depression is produced in the beam. Move the microscope downwards till its horizontal cross-wire again coincides with the previous division of the scale attached to the hook (or hanger). Take the readings of the vernier scale of the cathetometer. The difference between (4) and (5) gives the depression in the beam due to 1 kg weight.
6. Repeat the previous step for loads increasing by 1 kg at a time and then for loads decreasing in that order.
7. Measure the distance between knife edges (observation II)
8. Measure the breadth 'b' and thickness 'd' of the beam precisely using vernier callipers and screw gauge respectively (observation table III).
9. A graph is plotted between M and y.

From graph find the slope. calculate Y using the formula and slope of the graph.

Observation: (I) Readings for the depression 'y' in the beam due to the load applied:

Least count of vernier scale of cathetometer =

Zero error of the cathetometer =

S.No.	*Load applied M (in kg)*	*Reading (in cm) of cathetometer with*		*Mean of i and ii (in cm)*	*Depression produced in the beam for 3 kg of load y (in cm)*	*Mean depression produced in the beam for 3 kg of Load y (in cm)*
		Load increasing (i)	*Load decreasing (ii)*			
1	0					
2	1					
3	2					
4	3					
5	4					
6	5					

(II) The distance between two knife edges (l) =

(III) Readings for the breadth (b) and thickness (d) of the beam:

Least count of Vernier callipers =

Zero error of the vernier callipers =

Observation	1	2	3	4	5	6	7	8	9	10	Mean
Breadth b (in cm)											
Thickness d (in cm)											

Calculations: The Young's modulus of the material (Iron) of the beam can be calculated by substituting the values of M, y, L, b, d and g in C.G.S. in Eq.

$$Y = \frac{Mgl^3}{4ybd^3}$$

From graph $= \dfrac{gl^3}{4bd^3}\left(\dfrac{M}{y}\right)$

Result: The value of Young's modulus of the material (Iron) of the beam is found to be =
Standard Value =
Percentage error =

Precautions and Sources of Error

1. The beam must be symmetrically placed on the knife edges with equal lengths projecting out beyond the knife edges.
2. The hanger should be suspended from the centre of gravity of the beam.
3. The loads should be placed or removed from the hanger as gently as possible and the reading should be recorded only after waiting for sometime, so that the thermal effects produced in the specimen, get subsided.
4. Avoid the backlash error in the cathetometer.
5. Since the depth (thickness) of the beam appears as its cube in the formula and is relatively a small quantity, it should be determined by measuring it at several places along length by screw gauge.

Theoretical error: The value of Young's modulus for the material of the beam is given by

$$Y = \frac{Mgl^3}{4bd^3y}$$

Taking log and differentiating above expression, we get

$$\frac{\delta Y}{Y} = \frac{3\delta l}{l} + \frac{\delta b}{b} + \frac{3\delta d}{d} + \frac{\delta y}{y}$$

Maximum possible error = ________%.

3.24 OBJECT

To determine the Poisson's ratio for rubber.

Apparatus used: Rubber tube with metal sleeve and rubber stopper, metre scale, small pointer, slotted weights, hanger, Burette and rubber stopper

Formula used: The Poisson's ratio σ for rubber is given by

$$\sigma = \frac{1}{2}\left[1 - \frac{1}{A}\frac{dV}{dL}\right]$$

Where A = area of cross-section of rubber tube

$$= \pi \frac{D^2}{4} \text{ where } D \text{ is its diameter}$$

dV = Small increase in the volume of the tube when stretched by a small weight

dL = corresponding increase in the length of the tube.

Theory: Let a rubber tube suspended vertically be loaded at its lower end with a small weight. This stretches the rubber tube slightly with a consequent increase in its length and internal volume. Let V be the original volume of the tube, A its area of cross-section, L its length and D its diameter. Then, if for a small increase of volume dV, the corresponding increase in length is dL and the decrease in area is dA, we have

$$V + dV = (A - dA)(L + dL)$$
$$= AL + AdL - LdA - dAdL$$

Putting $AL = V$ and neglecting $dAdL$ a product of two small quantities, we have

$$dV = AdL - LdA \qquad ...(1)$$

Now $$A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$$

Differentiating $$dA = \frac{\pi D}{2} dD = \frac{2A}{D} dD$$

Substituting this value of dA in equation (1) we get

$$dV = AdL - \frac{2AL}{D} dD$$

or $$\frac{dV}{dL} = A - \frac{2AL}{D} \cdot \frac{dD}{dL}$$

Whence $$\frac{LdD}{DdL} = \frac{1}{2}\left(1 - \frac{dV}{AdL}\right)$$

Poisson's ratio $$\sigma = \frac{\frac{L}{dL}}{\frac{D}{dD}} = \frac{1}{2}\left(1 - \frac{1}{A}\frac{dV}{dL}\right)$$

This equation can be employed to calculate σ, if other quantities are determined.

Description of apparatus: A rubber tube about one metre long and 4 cms in diameter is suspended in a vertical position as shown in Fig. 3.10. Its two ends are closed by means of two rubber corks A and B such that both ends are water tight. A burette C about 50 cms long and 1 cm in diameter open at both ends is inserted in the rubber tube through the upper cork A. The tube is held vertical with most of its portion out of the rubber tube.

Water is filled in the rubber tube till it rises in the glass tube from the end A of rubber tube. A pointer P is fixed at lower end B which moves on a scale S when weights are placed on the hanger.

Procedure

1. The apparatus is suspended through a clamp fixed at a convenient height.
2. Pour water in the rubber tube until the water meniscus appears nearly at the top of the burette.
3. Note down the position of the pointer on the scale and the reading of water meniscus in burette. We take this as zero position though the hanger remains suspended from the hook.
4. Measure the diameter of the rubber tube at a number of points with a vernier callipers and find its mean value, and thus calculate the area of cross-section of the tube.
5. Place gently a load, of say 100 gm, on the hanger at the lower end of the tube and wait for about 5 minutes. Note down the readings of pointer and the meniscus of water.
6. The difference between the two readings of the burette gives the change in volume dV for the load on the hanger and the difference between the two readings of the pointer on the scale gives the corresponding change in length.
7. Now increase the load on the hanger in equal steps of, say 100 gm, till maximum permissible load within elastic limit is reached, taking down the reading of the burette and the pointer after addition of each load when the apparatus has settled down.
8. Repeat the above procedure for weights decreasing.
9. Take the mean of the two readings of the burette for the same load on the hanger obtained with increasing and decreasing load, and then subtracting the mean readings for zero load on the hanger from the mean reading for any load, calculate the change in volume dV of the rubber tube for various loads on the hanger.

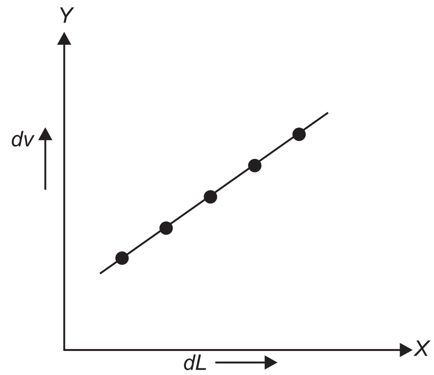


Fig. 3.10

10. Similarly calculate the volume of corresponding change in length dL for the various loads.
11. Calculate the value of $\frac{dV}{dL}$ for each set of observations separately and find its mean value for σ.
12. Plot a graph taking dL along the X-axis and the corresponding value of dV along the Y-axis. This will come out to be a straight line as shown in figure 3.10.

    Its slope will give the average value of $\frac{dV}{dL}$.
13. Also use this value to calculate again the value of σ.

Observation:

Least count of the scale =

Least count of microscope =

(A) Measurement of diameter of the rubber tube.

Vernier constant =

Zero error =

S.No.	Reading along any diameter α cm	Reading along perpendicular diameter β cm	Diameter $\left(\frac{\alpha+\beta}{2}\right)$ cm

Mean uncorrected diameter =

(B) Determination of change in volume *dV* and the corresponding change in length *dL*.

S.No.	Load on the hanger	Reading of burette			Change in volume dV	Reading of pointer P on scale S			Change in length dL	$\frac{dv}{dL}$	Mean
		Load increasing x	Load decreasing y	Mean $\left(\frac{x+y}{2}\right)$		Load increasing x'	Load decreasing y'	Mean $\frac{x'+y'}{2}$			

Calculations: Mean corrected diameter of the rubber tube = ... cm.

$\therefore$ Area of cross-section of the tube A = $\frac{\pi D^2}{4}$ = ... sq. cm

$$\sigma = \frac{1}{2}\left(1 - \frac{1}{A}\frac{dV}{dL}\right)$$

Also from graph calculate the value of slope $\frac{dV}{dL}$ and substitute this value in above equation.

Result: The value of Poisson's ratio for rubber as obtained experimentally

(*i*) by calculations =

(*ii*) by graph =

Standard value of σ =

∴ Error = %

Sources of error and precautions

1. Microscope should be used to measure internal radius of the rubber as also to measure the radius of the capillary.
2. Hanger should be stationary at the time of taking down the observations.
3. There should be no air bubble inside the rubber tube or the burette.
4. Weights should be placed or removed gently and in equal steps.
5. After each addition or removal of load wait for about 5 minutes before taking observations in order to allow the apparatus to settle down to new conditions of stress and strain.
6. The load suspended at the lower end of the rubber tube should not exceed the maximum load permissible within elastic limit.

3.25 FLY-WHEEL (MOMENT OF INERTIA)

Object: To determine the moment of inertia of a fly-wheel about its axis of rotation.

Apparatus: The flywheel, weight box, thread, stop-watch, meter scale and vernier callipers.

Description of apparatus: A fly-wheel is a heavy wheel or disc, capable of rotating about its axis. This fly-wheel properly supported in bearings may remain at rest in any position, *i.e.*, its centre of gravity lies on the axis of rotation. Its moment of inertia can be determined experimentally by setting it in motion with a known amount of energy.

Theory: The flywheel is mounted in its bearings with its axle horizontal and at a suitable height from the ground, and a string carrying a suitable mass m at its one end and having a length less than the height of the axle from the ground, is wrapped completely and evenly round the axle. When the mass m is released, the string unwinds itself, thus setting the flywheel in rotation. As the mass m descends further and further the rotation of the flywheel goes on increasing till it becomes maximum when the string leaves the axle and the mass drops off.

Let h be the distance fallen through by the mass before the string leaves the axle and the mass drops off, and let v and w be the linear velocity of the mass and angular velocity of the flywheel respectively at the instant the mass drops off. Then, as the mass descends a distance h, it loses potential energy mgh which is used up: (*i*) partly in providing kinetic energy of translation $\frac{1}{2}mv^2$ to the falling mass itself, (*ii*) partly in giving kinetic energy of rotation $\frac{1}{2}I\omega^2$ to the flywheel (where I is the moment of inertia of the flywheel about the axis of rotation) and (*iii*) partly in doing work against friction.

If the work done against friction is steady and F per turn, and, if the number of rotations made by the flywheel till the mass detaches is equal to n_1, the work done against friction is equal to n_1F. Hence by the principle of conservation of energy, we have

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + n_1F \qquad ...(1)$$

After the mass has detached the flywheel continues to rotate for a considerable time t before it is brought to rest by friction. If it makes n_2 rotations in this time, the work done against friction is equal to n_2F and evidently it is equal to the kinetic energy of the flywheel at the instant the mass drops off. Thus,

$$n_2F = \frac{1}{2} I\omega^2$$

$$F = \frac{1}{2}\frac{I\omega^2}{n_2} \quad ...(2)$$

Substituting this value of F in Eq. (1), we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}I\omega^2\frac{n_1}{n_2}$$

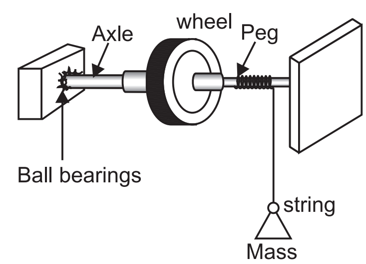


Fig. 3.11

Whence
$$\mathrm{I} = \frac{2mgh - mv^2}{\omega^2\left(1+\frac{n_1}{n_2}\right)}$$

If r be the radius of the flywheel,

$$v = r\omega$$

$\therefore$
$$\mathrm{I} = \frac{2mgh - mr^2\omega^2}{\omega^2\left(1+\frac{n_1}{n_2}\right)} \quad ...(3a)$$

$$= \frac{m\left(2\frac{gh}{\omega^2} - r^2\right)}{\left(1+\frac{n_1}{n_2}\right)} \quad ...(3b)$$

After the mass has detached, its angular velocity decreases on account of friction and after some time t, the flywheel finally comes to rest. At the time of detachment of the mass the angular velocity of the wheel is ω and when it comes to rest its angular velocity is zero. Hence, if the force of friction is steady, the motion of the flywheel is uniformly retarded and the average angular velocity during this interval is equal to $\frac{\omega}{2}$. Thus,

$$\frac{\omega}{2} = \frac{2\pi n_2}{t}$$

or
$$\omega = \frac{4\pi n_2}{t} \quad ...(4)$$

Thus observing the time 't' and counting the rotations n_1 and n_2 made by the flywheel its moment of inertia can be calculated from equation (3) and (4).

For a fly-wheel with large moment of inertia (I), $\frac{1}{2}mv^2$ may be neglected, the equation (3a) becomes

$$I = \frac{2mgh}{\omega^2\left(1+\frac{n_1}{n_2}\right)}$$

Substituting the value of ω from Eq. (4) in above equation, we get

$$I = \frac{mgh}{\dfrac{8\pi^2 n_2^2}{t^2}\left(1 + \dfrac{n_1}{n_2}\right)} \quad ..(5)$$

This formula is used to calculate the moment of inertia of a flywheel.

Procedure

1. Attach a mass *m* to one end of a thin thread and a loop is made at the other end which is fastened to the peg.
2. The thread is wrapped evenly round the axle of the wheel.
3. Allow the mass to descend slowly and count the number of revolutions n_1 during descent.
4. When the thread has unwound itself and detached from the axle after n_1 turns, start the stop watch. Count the number of revolutions n_2 before the flywheel comes to rest and stop the stop watch. Thus n_2 and t are known.
5. Repeat the experiment with three different masses.
6. Calculate the value of I using the given Eq. (5).

Observation: Least count of the stop-watch = ... sec.

Table for determination of n_1, n_2 and t

S.No.	Total load applied (in kg)	No. of revolutions of flywheel before the mass detached n_1	No. of revolutions of flywheel to come to rest after mass detached n_2	Time for n_2 revolutions t (sec)
1				
2				
3				

Calculation: $I = \frac{mgh}{\dfrac{8\pi^2 n_2^2}{t^2}\left(1 + \dfrac{n_1}{n_2}\right)}$

Result: The moment of inertia of the flywheel is ..... kg–m^2.

Sources of error and precautions

1. The length of the string should be always less than the height of the axle of the flywheel from the floor so that it may leave the axle before the mass strikes the floor.
2. The loop slipped over the peg should be quite loose so that when the string has unwound itself, it must leave the axle and there may be no tendency for it to rewind in the opposite direction.

3. The string should be evenly wound on the axle, *i.e.*, there should be no overlapping of, or a gap left between, the various coils of the string.
4. To ensure winding to whole number of turns of string on the axle the winding should be stopped, when almost complete at a stage where the projecting peg is horizontal.
5. To determine h measure only the length of the string between the loop and the mark at the other end where the string left the axle before the start of the flywheel.
6. The string used should be of very small diameter compared with the diameter of the axle. If the string is of appreciable thickness half of its thickness should be added to the radius of the axle to get the effective value of r.
7. The friction at the bearings should not be great and the mass tied to the end of the string should be sufficient to be able to overcome the bearing-friction and so to start falling of its own accord.
8. Take extra care to start the stop-watch immediately the string leaves the axle.

Criticism of the method: In this method the exact instant at which the mass drops off cannot be correctly found out and hence the values of n_1, n_2 and t cannot be determined very accurately. The angular velocity ω of the flywheel at the instant the mass drops off has been calculated from the formula $\omega = \frac{4\pi n_2}{t}$ on the assumption that the force of friction remains constant while the angular velocity of the flywheel decreases from ω to zero. But as the friction is less at greater velocities, we have no justification for this assumption. Hence for more accurate result, ω should be measured by a method in which no such assumption is made *e.g.*, with a tuning fork.

3.26 TORSION TABLE (ELASTICITY)

Object: To determine the modulus of rigidity of the material of the given wire and moment of inertia of an irregular body with the help of a torsion table.

Apparatus used: Torsion pendulum, a fairly thin and long wire of the material to be tested, clamps and chucks, a stop-watch, an auxillary body (a cylinder), screw guage, vernier callipers, meter scale, spirit level, a balance and a weight box.

Formula used:

$$\eta = \frac{8\pi I_1 l}{(T_1^2 - T_0^2) r^4}$$

where

$$I_1 = \frac{1}{2} MR^2$$

$$I_2 = \frac{T_2^2 - T_0^2}{T_1^2 - T_0^2} I_1$$

T_0 is the time period of oscillations of the torsion table only. T_1 is the time period of oscillations of the torsion table plus a regular body of moment of inertia I_1 placed on the table, its axis coinciding with the axis of the wire. T_2 is the time period for the torsion table plus an irregular body placed on the table. l is the length of the wire between two clamps, r is the radius of the wire, M is the mass of the regular cylinder, R is the radius of the regular cylinder.

Description of apparatus: The torsion table is illustrated in figure 3.12. One end of a fairly thin and long wire is clamped at A to a rigid support and the other end is fixed in the centre of a projection coming out of the central portion of a circular disc B, of aluminium or brass. On the upper face of the disc are described concentric circles and a concentric groove is cut in which three balancing weights can be placed. Beneath the disc is a heavy iron table T provided with three levelling screws. The plumb-line arrangement between the disc and the table serves to test the horizontality of the disc.

Theory: When the disc is rotated in a horizontal plane and then released, it executes torsional vibrations about the wire as the axis. If I_0 be the moment of inertia of the disc with its projection about the wire as the axis, its period of oscillation is, from eq.

$$T = 2\pi\sqrt{\frac{I}{C}} \text{ given by}$$

$$T_0 = 2\pi\sqrt{\frac{I_0}{C}} \qquad ...(1)$$

Where C is the torsional couple per unit radian twist. If an auxiliary body of known moment of inertia I is placed centrally upon the disc, then the period of oscillation T of the combination is given by

$$T = 2\pi\sqrt{\frac{I_0 + I}{C}} \qquad ...(2)$$

Squaring equations (1) and (2) and then subtracting equation (1) from Eq. (2), we get

$$T^2 - T^2{}_0 = \frac{4\pi^2 I}{C}$$

But from Eq. $\tau = \frac{\pi\eta r^4\phi}{2l}$, C is also-equal to $\frac{\pi\eta r^4}{2l}$. Hence the above equation yields.

$$\eta = \frac{8\pi I l}{(T^2 - T_0^2)r^4} \qquad ...(3)$$

From Eq. (3) the value of modulus of rigidity η of the wire can be calculated, if its length l and radius r are determined and the periods T_0 and T observed.

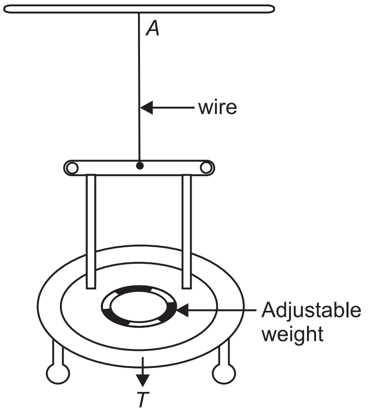


Fig. 3.12

Methods

1. Set up the torsion pendulum as shown in figure.
2. Level the heavy iron table T by the levelling screws and test the levelling with a spirit level.
3. Adjust the positions of the balancing weights in the groove in the disc such that the disc is horizontal as indicated by the plumb-line arrangement between the disc and the table. Place a vertical pointer in front of the disc and just behind it put a mark on the disc when the latter is at rest.

4. Rotate the disc slightly in a horizontal plane and then release it. It will perform torsional oscillations when undesirable motions have subsided, say after two or three oscillations, begin timing the oscillations of the disc with an accurate stop-watch by observing the transits of the mark on the disc past the vertical pointer. Determine twice the time for a large number of oscillations say, 30 and taking at least two sets of observations with different number of oscillations, find the mean period T_0 of the disc.
5. Place a right cylinder centrally upon the disc and determine as before, the period T of the combination. Note T will be greater than T_0.
6. Measure the length of the wire between the clamps with a metre scale and the diameter of the wire at several places with a screw gauge. Also weight the cylinder in a physical balance and find its diameter with vernier callipers.
7. Calculate the moment of inertia of the cylinder from the formula $I = MR^2/2$ and then the value of modulus of rigidity of the wire from Eq. (3).

Observation: (A) Determination of periods T and T_0

Least count of stop-watch = ... sec

S.No.	*No. of oscillation*	*Determination of T_0*			*Determination of T*		
		Time taken		*T_0 sec*	*Time taken*		*T sec*
		min	*sec*		*min*	*sec*	
			Mean T_0			Mean T	

(B) (*i*) Length of the wire = ...

(*ii*) Measurement of diameter of the wire

Least count of the screw gauge =

Zero error =

S.No.	*Reading along any diameter (a) c.m.*	*Reading along perpendicular diameter (b) c.m.*	*$\frac{a+b}{2}$(c.m.)*
		Mean uncorrected diameter	

(C) (*i*) Mass of the cylinder =
(*ii*) Measurement of diameter of the cylinder
Vernier constant of the callipers =
zero error =
Diameter of the cylinder—(*i*) ...... cm
(*ii*) ...... cm
(*iii*) ...... cm
(*iv*) ...... cm
Mean diameter = ... cm

Calculations:

Mean corrected diameter of the wire = ... cm
Radius of the wire = ... cm
Mean corrected diameter of the cylinder = cm
∴ Radius of the cylinder = ... cm

$$I = \frac{MR^2}{2}$$
$$= ......$$
$$\eta = \frac{8\pi Il}{(T^2 - T_0{}^2)\, r^4}$$

Result: The modulus of rigidity of the material of the given wire = ... dynes/cm^2
standard value = ... dynes/cm^2
∴ error = %

Sources of error and precautions:

1. The disc should always remain horizontal so that its moment of inertia I_0 remain unaltered throughout the experiment. Consequently the balancing weights, when once adjusted, should not be disturbed in subsequent observations for T.
2. The motion of the torsion pendulum should be purely rotational in a horizontal plane.
3. The suspension wire should be free from kinks and should be fairly thin and long, say about 70 cm, and 0.1 cm thick, so that torsional rigidity may be small and hence the periods of the pendulum large.
4. The wire should not be twisted beyond elastic limit otherwise the torsional couple will not be proportional to the value of the twist.
5. The auxiliary body (cylinder) should be of uniform density throughout e.g., of brass, and should be placed centrally upon disc so that its axis is coincident with the axis of the suspension wire.
6. As the periods occur raised to the second power in the expression for η, they must be measured very accurately by timing a large number of oscillations with a stop-watch reading upto $\frac{1}{5}$ sec.
7. As the radius of the wire occurs raised to the fourth power in the expression for η and is a small quantity, the diameter of the wire must be measured very accurately with a screw gauge. Reading should be taken at several points along the length of the wire and at each

point two mutually perpendicular diameters should be measured. The diameter of the cylinder should be similarly measured with a vernier callipers.

3.27 OBJECT

To determine the restoring force per unit extension of a spiral spring by statical and dynamical methods and also to determine the mass of the spring.

Apparatus used: A spiral spring, a pointer and a scale-pan, slotted weights and a stop watch.

Formula used: 1. Statical method

The restoring force per unit extensions (k) of the spring is given by

$$k = \frac{Mg}{l} \text{ Newton/meter}$$

where M = mass applied at the lower end of the spring

g = acceleration due to gravity

l = extension produced in the spring

2. Dynamical method

$$k = \frac{4\pi^2 (M_1 - M_2)}{(T_1^2 - T_2^2)} \text{ Newton/meter}$$

where M_1, M_2 = Masses applied at the lower end of the spring successively

T_1, T_2 = Time periods of the spring corresponding to masses M_1 and M_2 respectively.

3. The mass m of the spring is given by

$$m = 3\left[\frac{M_1 T_2^2 - M_2 T_1^2}{T_1^2 - T_2^2}\right] kg$$

where the symbols have their usual meanings.

Theory: A spiral-spring consists of a uniform wire, shaped permanently to have the form of a regular helix. A flat spiral-spring is one in which the plane of the spiral is perpendicular to axis of the cylinder. We deal here only with flat spiral-spring.

The spring is suspended from a rigid support with a hanger on the other end. A mass M is placed on the hanger so that spiral spring stretched vertically downward. If the extension of the spring is small, the force of elastic reaction F is, from Hooke's law, proportional to the extension l, or

$$F \propto l$$

or $$F = kl$$

where k is a constant giving a measure of the stiffness of the spring and is called the restoring force per unit extension of the spring.

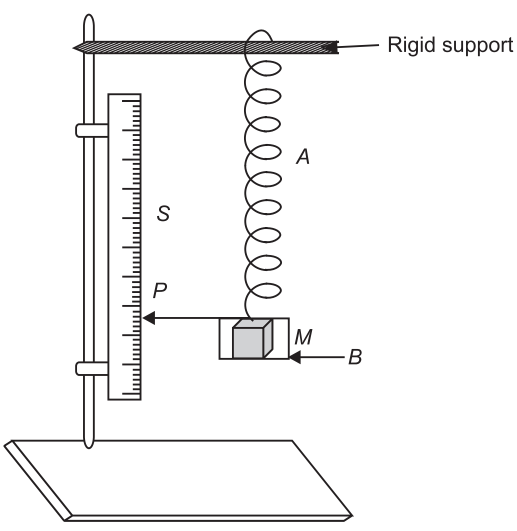


Fig. 3.13

But in the equilibrium state

$$F = Mg$$

$$\therefore \qquad kl = Mg$$

or
$$\boxed{k = \frac{Mg}{l}}$$

from which k may be calculated, if the extension l for a known load M at the end of the spring is determined.

If the mass at the end of the spring is now displaced vertically downward and then released, then for small oscillations, the restoring force at any instant is proportional to the displacement x, *i.e.*, $F = kx$.

If $\frac{d^2x}{dt^2}$ is the acceleration of the mass at that instant, the inertial reaction of the system is $(M+m)\frac{d^2x}{dt^2}$, where m is the effective mass of the spring. Equating the sum of these forces to zero we get from Newton's third law of motion, the equation

$$(M+m)\frac{d^2x}{dt^2} + kx = 0$$

or
$$\frac{d^2x}{dt^2} + \frac{k}{(M+m)}x = 0$$

This equation represents a S.H.M. whose period is given by

$$T = 2\pi\sqrt{\frac{M+m}{k}} \qquad ...(1)$$

If the experiment is performed with two masses M_1 and M_2 suspended successively at the end of the spring and the respective periods are T_1 and T_2, we have

$$T_1 = 2\pi\sqrt{\frac{M_1+m}{k}} \qquad ...(2)$$

and
$$T_2 = 2\pi\sqrt{\frac{M_2+m}{k}} \qquad ...(3)$$

Squaring Eq. (2) and (3) and subtracting, we get

$$T_1^2 - T_2^2 = \frac{4\pi^2}{k}(M_1 - M_2)$$

Whence
$$\boxed{k = \frac{4\pi^2(M_1 - M_2)}{T_1^2 - T_2^2}} \qquad ...(4)$$

From which k may be calculated.

Squaring Eq. (2) and (3) and then dividing Eq. (2) by Eq. (3) we have

$$\frac{T_1^2}{T_2^2} = \frac{M_1 + m}{M_2 + m}$$

Whence on simplification

$$m = \frac{M_1 T_2^2 - M_2 T_1^2}{T_1^2 - T_2^2}$$

It can be shown that the mass of the spring m' is three times the effective mass of the spring.

$\therefore$ Mass of the spring $m' = 3\left[\frac{M_1 T_2^2 - M_2 T_1^2}{T_1^2 - T_2^2}\right]$ ...(5)

Squaring and rearranging Eq. (1) gives

$$T^2 = \left(\frac{4\pi^2}{k}\right)M + \left(\frac{4\pi^2}{k}\right)m \qquad \text{...(6)}$$

A plot of T^2 against M gives a straight line as shown in figure 3.14. The negative intercept OP on the x-axis equals m and may be used to find m graphically.

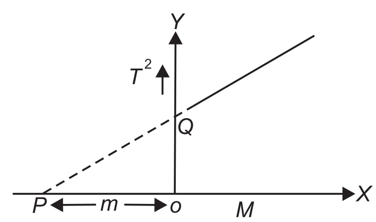


Fig. 3.14

Description of apparatus: A spiral spring A whose restoring force per unit extension is to be determined is suspended from a rigid support as shown in figure 3.13. At the lower end of the spring, a small scale pan is fastened. A small horizontal pointer P is also attached to the scale pan. A millimeter scale S is also set in front of the spring in such a way that when spring vibrates up and down, the pointer freely moves over the scale.

Procedure: (*a*) *Statical Method*

1. Hang a spiral spring A from a rigid support as shown in figure 3.13 and attached a scale pan B.
2. With no load in the scale-pan, note down the zero reading of the pointer on the scale.
3. Place gently in the pan a load of, say 100 gm.
4. Now the spring slightly stretches and the pointer moves down on the scale. In the steady position, note down the reading of the pointer. The difference of the two readings is the extension of the spring for the load in the pan.
5. Increase the lead in the pan in equal steps until maximum permissible load is reached and note down the corresponding pointer readings on the scale.
6. The experiment is repeated with decreasing loads.
7. Plot a graph as illustrated in figure 3.15 between the load and the scale readings taking the load on X-axis and corresponding scale readings on Y-axis. The graph will be a straight line. Measure PR and QR and calculate k from the formula $k = \frac{QR}{PR} \times g$ Newton/meter.

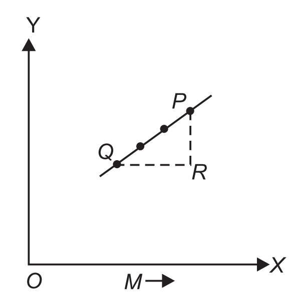


Fig. 3.15

Observation: Mass of the pan =

Table for the extension of the spiral-spring

S.No.	*Load in the pan*	*Reading of pointer on the scale in cm*			*Extension for ...... gms (in cm)*
		Load increasing	*Load decreasing*	*Mean*	

Average extension for .... kgs load = ... cm.

Calculations: Statical experiment

The restoring force per unit extension of the spring

$$k = \frac{mg}{l} = \text{......} \text{ Newton/meter}$$

A graph is plotted between the load and scale readings described in point (7) in the procedure. From graph

$$PR = \text{.....}$$
$$QR = \text{.....}$$

$$\therefore \qquad k = \frac{QR}{PR} \times g = \text{.......} \text{ Newton/meter}$$

Result: The restoring force per unit extension of the spring = ... Newton/meter

The mass of the spring = ... gm

Sources of errors and precautions

1. The axis of the spiral spring must be vertical.
2. The scale should be set up vertically and should be arranged to give almost the maximum extension allowed.
3. The pointer should move freely over the scale and should be just not in contact with it.
4. The spiral spring should not be stretched beyond elastic limit.
5. The load in the scale-pan should be placed gently and should be increased in equal steps.
6. While calculating the mean extension of the spring for a certain load, successive difference between consecutive readings of the pointer on the scale should not be taken.

Procedure *(dynamical experiment)*

1. Load the pan. Displace the pan vertically downward through a small distance and release it. The spring performs simple harmonic oscillations.

2. With the help of stop watch, note down the time of a number of oscillation (say 20 or 30). Now divide the total time by the number of oscillations to find the time period (time for one oscillation) T_1.
3. Increase the load in the pan to M_2. As described above, find the time period T_2 for this load.
4. Repeat the experiment with different values of load.
5. Plot a graph between T^2 and M. The slope is $\frac{4\pi^2}{k}$ and the intercept on the negative side of the x-axis is m.

Observations: Measurement of periods T_1 and T_2 for the loads M_1 and M_2. Least count of stop-watch = sec.

S.No.	*Load in the pan*		*No. of Oscillations*	*Time taken with load*		*Periods*		*K Newton/ sec*	*m*
	M_1	*M_2*		*M_1*	*M_2*	*T1*	*T_2*		
	gms	*gms*		*min sec*	*min sec*				
								Mean	

Calculations: Restoring force per unit extension of the spring

$$k = \frac{4\pi^2 (M_1 - M_2)}{T_1^2 - T_2^2}$$

$$= \text{.....} \text{ Newton/metre}$$

$$\text{Mass of the spiral spring } m' = 3\left[\frac{M_1T_2^2 - M_2T_1^2}{T_1^2 - T_2^2}\right]$$

$$= \text{....} \text{ gm}$$

Result: The restoring force per unit extension of the spring as determined by dynamical experiment = ... Newton/m

The mass of spring = ... gm

Sources of error and precautions

1. The spiral spring should oscillate vertically.
2. The amplitude of oscillation should be small.

3. As in the expression for k and m' the periods T_1 and T_2 occur raised to the second power, they should be accurately determined by timing a large number of oscillations correct upto the value measurable with the stop-watch.

3.28 OBJECT

To study the oscillations of a rubber band and a spring.

Apparatus used: Rubber bands (cycle tube), a pan, mounting arrangement, weight of 50 gm, stop watch and spring.

Formula used: (1) For experimental verification of formula

$$\frac{T_1}{T_2} = \sqrt{\left(\frac{m_1 g}{m_2 g}\right)} \times \sqrt{\frac{k_{x'_0}}{k_{x_0}}}$$

where T_1 = Time period of a rubber band when subjected to a load $m_1 g$.

T_2 = Time period of the same rubber when subjected to a load $m_2 g$.

k_{x_0} = force constant of rubber band corresponding to equilibrium extension x_0.

$k_{x'_0}$ = force constant of rubber band corresponding to equilibrium extension x'_0.

Here x_0 and x'_0 are the equilibrium extensions corresponding to loads $m_1 g$ and $m_2 g$.

(2) The entire potential energy U (joule) of the system is given by

$$U = U_b - mg \cdot x$$

where U_b = potential energy of the rubber band or springs.

x = displacement from the equilibrium position due to a load mg.

$-mg\,x$ = gravitational energy of mass m which is commonly taken as negative.

Procedure:

1. Set up the experimental arrangement as shown in figure 3.16 in such a way that when a load is subjected to the rubber band, the pointer moves freely on metre scale. Remove the load and note down the pointer's reading on metre scale when rubber band is stationary.
2. Place a weight of 0.05 kg on the pan. Now the rubber band is stretched. Note down the pointer reading on the meter scale.
3. Continue the process (2) of loading the rubber band in steps of 0.05 kg and noting the extension within the elastic limit.
4. The reading of the pointer is also recorded by removing the weights in steps. If the previous readings are almost repeated then the elastic limit has not exceeded. For a particular weight, the mean of the corresponding readings gives the extension for that weight.
5. Again place 0.05 kg in the pan and wait till the pointer is stationary. Now slightly pull down the pan and release it. The pan oscillates vertically with amplitude decreasing pretty quickly. Record the time of few oscillations with the help of sensitive stop watch. Calculate the time period. Repeat the experiment for other loads to obtain the corresponding time period.
6. Draw a graph between load and corresponding extension. The graph is shown in figure 3.17. Take different points on the curve and draw tangents. Obtain the values of Δm and

Δx for different tangents. Calculate the force constant using the following formula.

$$k_{x_0} = g\left(\frac{\Delta m}{\Delta x}\right)_{x_0}$$

Record the extensions from graph and corresponding force constants in the table.

7. Calculate the time periods by using the formula

$$T_1 = 2\pi\sqrt{\frac{m_1}{k_{x_0}}}$$

and $$T_2 = 2\pi\sqrt{\frac{m_2}{k_{x_0'}}}$$

Compare the experimental time periods with calculated time periods.

8. From load extension graph, consider the area enclosed between the curve and the extension axis for different load increasing in regular steps. The areas are shown in figure 3.18. The area gives U_b corresponding to a particular extension.

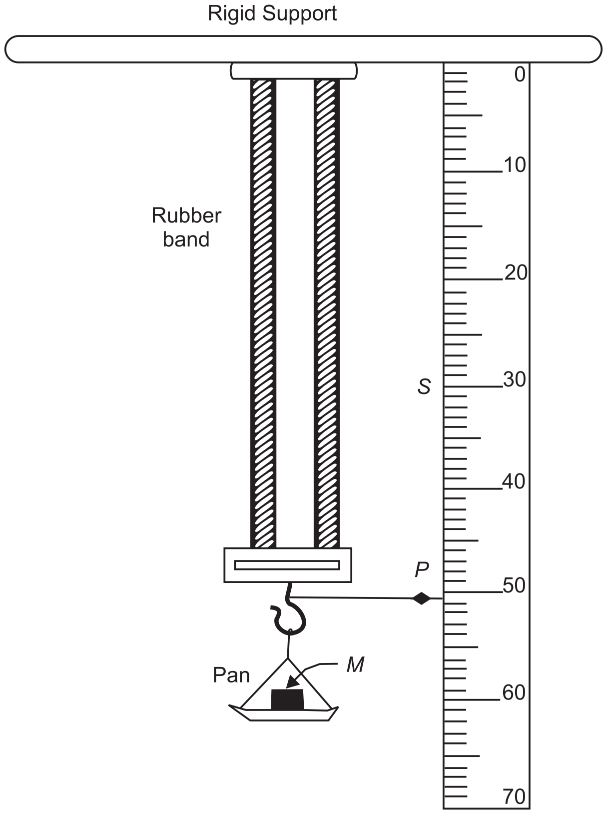


Fig. 3.16

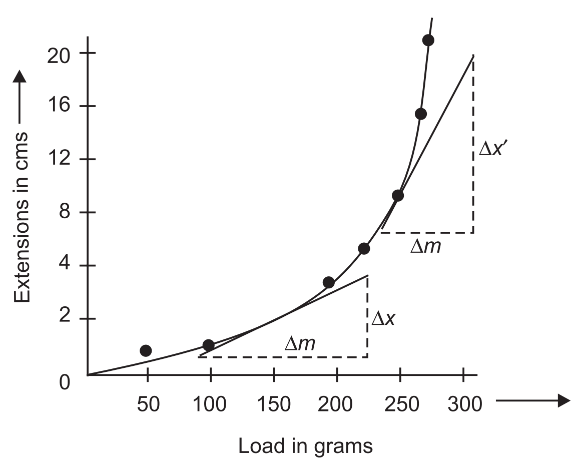


Fig. 3.17

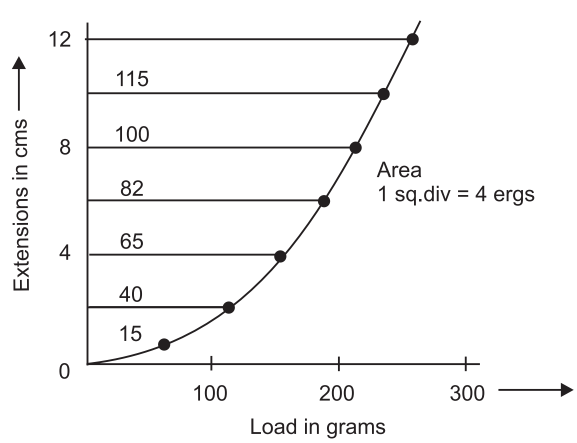


Fig. 3.18

9. Calculate U_m for mass = 100 g and obtain the value of U by the formula
$$= U_b + U_m$$

10. Draw a graph in extension and the corresponding energies i.e. U_b, U_m and U. The graph is shown in Fig. 3.19.
11. Same procedure can be adopted in case of a spring.

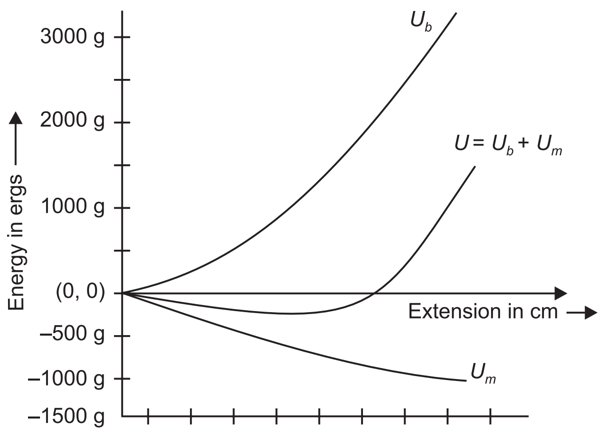


Fig. 3.19

Observations: (I) For load extension graph

S. No.	*Mass suspended in gm*	*Reading of pointer with load*		*Mean (a + b)/2 (meter)*	*Extension in rubber band (meter)*
		Increasing (a) meter	*Decreasing (b) meter*		
1. 2. 3. 4. 5. 6. 7. 8.					

Original length of the rubber band = ... cm

(II) For oscillations of the band

S. No.	*Mass Suspended in gm*	*No. of oscillations*	*Time sec*	*Time period (observed) from graphs*	*Eq. Ex. tension*	*k from graph*	*Period (Cal.)*
1. 2. 3. 4.							

Calculations: From Graph

$$k_{x_0} = g\left(\frac{\Delta m}{\Delta x}\right)_{x_0} = ...$$

$$k_{x'_0} = g\left(\frac{\Delta m}{\Delta x'}\right)_{x'_0} = ...$$

$$T_1 = 2\pi\sqrt{\frac{m_1}{k_{x_0}}} = ...\text{ second}$$

$$T_2 = 2\pi\sqrt{\frac{m_2}{k_{x'_0}}} = \text{second.}$$

Results:

1. The force constant of rubber band is a function of extension *a* in elastic limit.
   If the same experiment is performed with spring, then it is observed that the force constant is independent of extension *a* within elastic limit.
2. From table (II), it is observed that the calculated time periods are the same as experimentally observed time periods.
3. U_b, U_m and U versus extension are drawn in the graphs.

Sources of error and precautions:

1. The rubber band should not be loaded beyond 8% of the load required for exceeding the elastic limit.
2. Time period should be recorded with sensitive stop watch.
3. The experiment should also be performed by decreasing loads.
4. The experiment should be performed with a number of rubber bands.
5. Amplitude of oscillations should be small.
6. For graphs, smooth waves should be drawn.

3.29 OBJECT

To determine Young's Modulus, Modulus of rigidity and Poisson's ratio of the material of a given wire by Searle's dynamical method.

Apparatus used: Two identical bars, given wire, stop watch, screw-gauge, vernier callipers, balance, candle and match box.

Formula used: The Young's Modulus (Y), modulus of rigidity (η) and Poisson's ratio (σ) are given by the formula,

$$Y = \frac{8\pi I l}{T_1^2 r^4}\ \text{Newton/meter}^2$$

$$\eta = \frac{8\pi I l}{T_2^2 r^4}\ \text{Newton/meter}^2$$

$$\sigma = \frac{T_2^2}{2T_1^2} - 1$$

Description of the apparatus: Two identical rods *AB* and *CD* of square or circular cross section connected together at their middle points by the specimen wire, are suspended by two silk fibres from a rigid support such that the plane passing through these rods and wire is horizontal as shown in figure 3.20.

Theory: Two equal inertia bars *AB* and *CD* of square section are joined at their centres by a fairly short and moderately thin wire *GG′* of the material whose elastic constants are to be determined, and the system is suspended by two parallel torsionless threads, so that in the equilibrium position the bars may be parallel to each other with the plane *ABDC* horizontal. If the two bars be turned through equal angles in opposite directions and be then set free, the bars will execute flexural vibrations in a horizontal plane with the same period about their supporting threads.

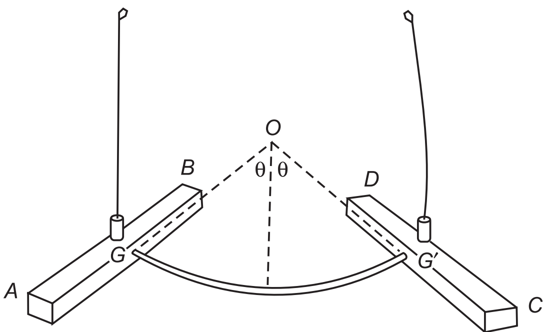


Fig. 3.20

When the amplitude of vibration is small, the wire is only slightly bent and the distance *GG′* between the ends of the wire measured along the straight line will never differ perceptibly from the length of the wire so that the distance between the lower ends of the supporting threads remains practically constant and hence the threads remain vertical during the oscillations of the bars, and there is thus no horizontal component of tensions in the threads acting on the wire.

The mass of the wire is negligible compared with that of the bars so that the motion of *G* and *G′* at right angles to *GG′* may be neglected. Further, since the horizontal displacement of *G* and *G′* are very small compared with the lengths of the supporting threads, the vertical motion of *G* and *G′* is also negligible. The centres of gravity of the bars, therefore, remain at rest, and hence the action of the wire on either bar and vice versa is simply a couple which, by symmetry must have a vertical axis. The moment of this couple called the "bending moment" is the same at every point of the wire and thus the neutral filament of the wire is bent into a circular arc.

If ρ is the radius of the arc, *Y* the Young's modulus for the material of the wire and *l* the geometrical moment of inertia of the area of cross-section of the wire about an axis through the centroid of the area and perpendicular to the plane of bending, the bending moment is from equation $G = \frac{YI}{\rho}$. If *l* is the length of the wire and θ the angle turned through by either bar, $\rho = \frac{l}{2\theta}$ and $G = \frac{2YI\theta}{l}$; and if $\frac{d^2\theta}{dt^2}$ is the angular acceleration of each bar towards its equilibrium position and *K* the moment of inertia of the bar about a vertical axis through its C.G., the torque due to inertial reaction is $K\frac{d^2\theta}{dt^2}$. Hence equating the sum of these two torques to zero, we get, from Newton's third law, the equation

$$K\frac{d^2\theta}{dt^2} + \frac{2YI\theta}{l} = 0$$

or
$$\frac{d^2\theta}{dt^2} + 2\frac{YI\theta}{Kl} = 0$$

The motion of the bars is, therefore, simple harmonic and hence the period of the flexural vibrations is given by

$$T_1 = 2\pi\sqrt{\frac{Kl}{2YI}}$$

whence
$$y = \frac{2\pi^2 Kl}{T_1^2 I}$$

If the radius of the wire be r, $I = \frac{1}{4}\pi r^4$ and hence

$$y = \frac{8\pi Kl}{T_1^2 r^4} \qquad ...(1)$$

If the length and breadth (horizontal) of the bar be a and b respectively,

$$K = M\left(\frac{a^2 + b^2}{12}\right)$$

where M = mass of the bar.

Now the suspensions of the bars are removed and one of the bars is fixed horizontally on a suitable support, while the other is suspended from a vertical wire. If the wire is twisted through an angle and the bar allowed to execute torsional oscillations, the period of oscillations is given by

$$T_2 = 2\pi\sqrt{\frac{K}{C}}$$

where C is the restoring couple per unit radian twist due to torsional reaction of the wire and is equal to $\pi\eta\, r^4/2l$, where η is the modulus of rigidity for the material of the wire. Thus

$$C = \frac{\pi\eta r^4}{2l} = \frac{4\pi^2 K}{T_2^2}$$

whence
$$\eta = \frac{8\pi Kl}{T_2^2\, r^4} \qquad ...(2)$$

Dividing Eq. (1) by Eq. (2), we get

$$\frac{Y}{\eta} = \frac{T_2^2}{T_1^2}$$

Now $Y = 2\eta\,(1 + \sigma)$, where σ is the Poisson's ratio. Hence

$$\sigma = \frac{Y}{2\eta} - 1$$

or

$$\sigma = \frac{T_2^2}{2T_1^2} - 1$$

Procedure:

1. Weigh both the bars and find the Mass M of each bar.
2. The breadth '*b*' of the cross bar is measured with the help of vernier callipers. If the rod is of circular cross-section then measure its diameter D with vernier callipers.
3. Measure the length L of the bar with an ordinary meter scale.
4. Attach the experimental wire to the middle points of the bar and suspend the bars from a rigid support with the help of equal threads such that the system is in a horizontal plane Fig. 3.21(*a*).
5. Bring the two bars close together (through a small angle) with the help of a small loop of the thread as shown in Fig. 3.21(*b*).
6. Burn the thread. Note the time period T_1 in this case.
7. Clamp one bar rigidly in a horizontal position so that the other hangs by the wire Fig. 3.21(*c*). Rotate the free bar through a small angle and note the time period T_2 for this case also.

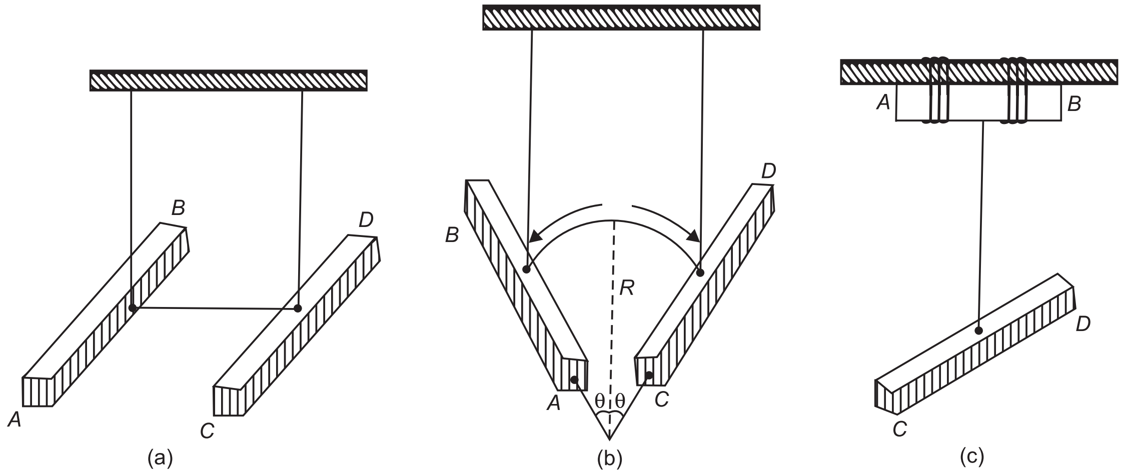


Fig. 3.21

8. Measure the length l of the wire between the two bars with meter scale.
9. Measure the diameter of the experimental wire at a large number of points in a mutually perpendicular directions by a screw gauge. Find r.

Observations: (A) Table for the determination of T_1 and T_2.
Least count of the stop-watch = ... secs.

S.No.	No. of oscillations (n)	Time		T_1	Time period $T_1 = (a/n)$ sec	Mean T_1 secs	Time		T_2	Time period $T_2 = (b/n)$ secs	Mean T_2 secs
		Min	Secs	Total secs (a)			Min	Secs	Total secs (b)		
1.	10										
2.	15										
3.	20										
4.	25										
5.	30										
6.	35										

(B) Mass of either of the AB or CD rod = ... gms = ... kg.
(C) Length of the either bar (L) = ... cms.
(D) Table for the measurement of breadth of the given bar.

$$\text{Least count of vernier callipers} = \frac{\text{Value of one div. of main scale in cm}}{\text{Total no. of divisions on vernier scale}} = \text{...... cm.}$$

Zero error of vernier callipers = ± ...... cms.

S. No.	Reading along any direction (→)			Reading along a perpendicular direction (↑)			Uncorrected breadth	Mean corrected breadth b cm
	M.S.	V.S.	Total x–cm	M.S.	V.S.	Total y–cm	$b = \frac{(x+y)}{2}$ cm	
1.								
2.								
3.								
4.								
5.								
6.								

b = ... cm = ... meter

If the bars are of circular cross-section then the above table may be used to determine the diameter D of the rod.

(E) Length (l) of wire = cms.
(F) Table for the measurement of diameter of the given wire.

$$\text{Least count of screw gauge} = \frac{\text{Pitch}}{\text{total no. of divisions on circular scale}} = \text{...... cm.}$$

Zero error of screw gauge = ± ...... cms.

S. No.	Reading along any Direction (→)			Reading along a perpendicular direction (↑)			Uncorrected diameter (X + Y)/2 cm	Mean uncorrected diameter cm	Mean corrected diameter (d) cm	Mean radius r = d/2 cm
	M.S. reading	V.S. reading	Total X–cm	M.S. reading	V.S. reading	Total Y–cm				
1. 2. 3. 4. 5. 6.										

r = ... cms = ... meter

N.B.: Record the mass and dimensions of the second inertia bar also, if the two bars are not exactly identical.

Calculations:

$$I = \frac{M\,(L^2 + b^2)}{12} = \text{......} \text{ kg} \times m^2 \text{ (for square cross-section bar)}$$

$$I = M\left(\frac{L^2}{12} + \frac{d^2}{16}\right) = \text{......} \text{ kg} \times m^2 \text{ (for circular bar)}$$

(*i*) $Y = \dfrac{8\pi Il}{T_1^2 r^4}$ = ... Newton/meter^2

(*ii*) $\eta = \dfrac{8\pi Il}{T_2^2 r^4}$ = ... Newton/meter^2

(*iii*) $\sigma = \dfrac{T_2^2}{2T_1^2} - 1$ = ...

Results: The values of elastic constants for the material of the wire are

Y = ... Newton/meter^2
η = ... Newton/meter^2
and σ = ...

Standards Results:

Y = ... Newton/meter^2
η = ... Newton/meter^2
and σ = ...

Percentage errors:

Y = ... %
η = ... %
and σ = ... %

Precautions and sources of error:

1. Bars should oscillate in a horizontal plane.
2. The amplitude of oscillations should be small.
3. The two bars should be identical.

4. Length of the two threads should be same.
5. Radius of wire should be measured very accurately.

Theoretical error:

$$Y = \frac{8\pi Il}{T_1^2 r^4} = \frac{8\pi l}{T_1^2 \left(\frac{d}{2}\right)^4} \times \frac{M\left(L^2 + b^2\right)}{12}$$

Taking log and differentiating

$$\frac{\delta Y}{Y} = \frac{\delta l}{l} + \frac{\delta M}{M} + \frac{2L\,\delta L}{\left(\frac{L^2 + b^2}{12}\right)} + \frac{2b\,\delta b}{\left(\frac{L^2 + b^2}{12}\right)} + \frac{2\delta T_1}{T_1} + \frac{4\delta d}{d}$$

Maximum possible error = ... %

Similarly find it for η and σ.

3.30 OBJECT

To determine the value of the modulus of rigidity of the material of a given wire by a dynamical method using Maxwell's needle.

Apparatus used: Maxwell's needle, screw gauge, given wire, meter scale, stop watch, physical balance and weight box.

Formula used: The modulus of rigidity η of the material of the wire is given by

$$\eta = \frac{2\pi l\,(M_S - M_H)\,L^2}{r^4\,(T_2^2 - T_1^2)} \text{ Newton/meter}^2$$

where l = length of the experimental wire
L = length of the brass tube
r = radius of the wire
M_S = mass of each of the solid cylinder
M_H = mass of each of the hollow cylinder
T_1 = time period when solid cylinders are placed in the middle
T_2 = time period when hollow cylinders are placed in the middle

Description of apparatus: Maxwell's needle is shown in figure 3.22. It consists of a hollow cylindrical brass tube of length L, suspended by a wire whose modulus of rigidity is to be determined. The tube is open at both ends. The hollow tube is fitted with four brass cylinders, two solids SS and two hollow HH, each having a length $\frac{L}{4}$ and same radii. These cylinders are inserted in the hollow tube symmetrically so that either solid cylinders SS are inside and hollow HH outside or hollow one inside and solid cylinder outside. A mirror M is attached to the wire for counting vibrations with lamp and scale arrangement.

Theory: Let the two hollow cylinders be placed in the middle and the solid ones at the two ends of the tube and let the combination be slightly rotated in a horizontal plane and then

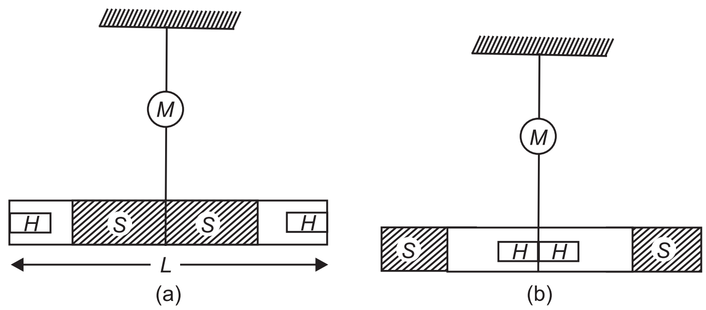


Fig. 3.22

released. The body will then execute S.H.M., about the wire as the axis and the period of oscillation is given by

$$T_1 = 2\pi \sqrt{\frac{I_1}{C}} \qquad ...(1)$$

where I_1 is the moment of inertia of combination about the wire as the axis and C the restoring couple per unit twist due to torsional reaction.

Now let the positions of the hollow and solid cylinders be interchanged so that the solid cylinder are now in the middle. Then, if I_2 is the moment of inertia of the new combination about the axis of rotation, the new period of oscillation is given by

$$T_2 = 2\pi \sqrt{\frac{I_2}{C}} \qquad ...(2)$$

Squaring equation (1) and (2) and subtracting eqn. (2) from eqn. (1), we get

$$T_1^2 - T_2^2 = \frac{4\pi^2}{C}(I_1 - I_2)$$

whence $$C = \frac{4\pi^2 (I_1 - I_2)}{T_1^2 - T_2^2}$$

But from eq. $C = \frac{\pi\eta r^4}{2l}$ where r is the radius and l the length of the wire whose modulus of rigidity is η.

Hence $$\frac{\pi\eta r^4}{2l} = \frac{4\pi^2 (I_1 - I_2)}{T_1^2 - T_2^2}$$

or $$\eta = \frac{8\pi l (I_1 - I_2)}{r^4 (T_1^2 - T_2^2)} \qquad ...(3)$$

Now let m_1 and m_2 be the masses of each of the hollow and the solid cylinders respectively and I_0, I' and I'' be the moments of inertia of the hollow tube, the hollow cylinder and the solid cylinder respectively about a vertical axis passing through their middle points. Then, if L is the length of the hollow tube,

$$I_1 = I_0 + 2I' + 2m_1 \left(\frac{L}{8}\right)^2 + 2I'' + 2m_2 \left(\frac{3L}{8}\right)^2$$

and

$$I_2 = I_0 + 2I'' + 2m_2 \left(\frac{L}{8}\right)^2 + 2I' + 2m_1 \left(\frac{3L}{8}\right)^2$$

Hence

$$I_1 - I_2 = 2m_2 \left[\left(\frac{3L}{8}\right)^2 - \left(\frac{L}{8}\right)^2\right] - 2m_1 \left[\left(\frac{3L}{8}\right)^2 - \left(\frac{L}{8}\right)^2\right] = (m_2 - m_1)\,\frac{L^2}{4}$$

Putting this value of $(I_1 - I_2)$ in equation (3) we get

$$\boxed{\eta = \frac{2\pi l\,(m_2 - m_1)\,L^2}{r^4\,(T_1^2 - T_2^2)}}$$

This expression can be used to find the value of modulus of rigidity η of the material of the suspension wire.

Procedure:

1. Suspend the Maxwell's needle from the lower end of a thin and long wire of experimental material and fix the upper end to a rigid support.
2. By slightly rotating the Maxwell's needle about the wire in the horizontal plane, it is allowed to perform torsional oscillations. Keep the solid cylinders inside. Start the stop watch and simultaneously count the number of oscillations. In this way determine the time period of 10 oscillations. Similarly, obtain time periods for 20, 30, 40 and 50 oscillations and evaluate the mean value of time period T_1.
3. Interchange the positions of the cylinders (hollow cylinders inside) and find out the value of the time period T_2 for the system in this case. You will find that $T_2 > T_1$.
4. Measure the diameter of the wire at a large number of points in two mutually perpendicular directions at each point. Measure also the length l of the wire.
5. Using a meter scale, measure length L of the tube. Determine values of $(M_S - M_H)$ with the help of a physical balance.

Observations: (I) Table for the determination of T_1 and T_2.
Least count of the stop watch = ... secs.

S. No.	*No. of ocillations*	*Solid cylinders on the inner side*					*Solid cylinder on the outer side*				
		Time taken			*Time period T_1 secs*	*Mean T_1 secs*	*Time taken*			*Time period T_2 secs*	*Mean T_2 secs*
		Min	*Secs*	*Total secs*			*Min*	*Secs*	*Total secs*		
1.											
2.											
3.											
4.											
5.											
6.											

(II) Table for the determination of radius 'r' of the wire

Least count of screw gauge = $\frac{\text{Pitch}}{\text{Total no. of divisions on circular scale}}$ = ... cm

Zero error of screw gauge = ± ...... cm

S. No.	*Reading along any direction (→)*			*Reading along a perpendicular direction (↑)*			*Uncorrected diameter (X + Y)/2 cm*	*Mean Uncorrected diameter cms*	*Mean corrected diameter (d) cms*	*Mean radius r = (d/2) cm*
	M.S.	*V.S.*	*Total X–cms*	*M.S.*	*V.S.*	*Total Y–cms*				

r = ... cm = ... meter

(III) Mean mass of a hollow cylinder M_H = ... kg.
(IV) Mean mass of solid cylinder M_S = ... kg.
(V) Length of the Maxwell's needle L = ... meter
(VI) Length of the wire l = ... meter

Calculations: Modulus of rigidity η is calculated by using the following formula:

$$\eta = \frac{2\pi l\,(M_S - M_H)\,L^2}{r^4\,(T_1^2 - T_2^2)} \text{ Newton/meter}^2$$

Result: The modulus of rigidity of the material of the wire (.....) as found experimentally
= ...... Newton/meter^2

Standard value: Standard Value of η for ...... = ... Newton/meter^2
Percentage error = ... %

Precautions and sources of error:

1. The two sets of cylinders should be exactly identical and the hollow tube should be clamped exactly in the middle.
2. The Maxwell's needle should always remain horizontal so that the moment of inertia of the hollow tube about the axis of rotation remains unaltered throughout the whole

experiment. Hence while placing the cylinders inside the tube, no portion of them should be left projecting outside the hollow tube.

3. The motion of the Maxwell's needle should be purely rotational in a horizontal plane. All undesirable motions (up and down, or pendular) should be competely checked.
4. As in the expression for η the periods occur raised to the second power, they must be carefully measured by timing a large number of oscillations with an accurate stop-watch up to an accuracy of say, $\frac{1}{5}$ of a second.
5. The wire should not be twisted beyond elastic limit otherwise the restoring couple due to torsional reaction will not be proportional to value of the twist.
6. There should be no kinks in the wire. The wire should be fairly long and thin particularly when the rigidity is high so that the restoring couple per unit twist due to torsional reaction may be small and hence the period of oscillation of the Maxwell's needle is large.
7. In the expression for η the radius occurs raised to the fourth power and is a very small quantity usually of the order of 0.1 cm. Hence the diameter must be measured very accurately. Readings should be taken at several points equally spaced along the wire and two diameters at right angles to each other should be measured at each point, care being taken not to compress the wire in taking the readings.

3.31 OBJECT

To study the variation of moment of inertia of a system with the variation in the distribution of mass and hence to verify the theorem of parallel axes.

Apparatus used: Maxwell's needle apparatus with solid cylinders only and a stop watch or a light aluminium channel about 1.5 metre in length and 5 cm in breadth fitted with a clamp at the centre to suspend it horizontally by means of wire, two similar weights, stop watch and a metre scale.

Formula used: The time period T of the torsional oscillations of the system is given by

$$T = 2\pi\sqrt{\left[\frac{I_0 + 2I_S + 2m_S x^2}{C}\right]}$$

where I_0 = moment of inertia of hollow tube or suspension system.

I_S = moment of inertia of solid cylinder or added weight about an axis passing through their centre of gravity and perpendicular to their lengths.

m_S = mass of each solid cylinder or each added weight

x = distance of each solid cylinder or each added weight from the axis of suspension.

C = torsional rigidity of suspension wire.

Squaring the above equation

$$T^2 = \frac{4\pi^2}{C}\left[I_0 + 2I_S + 2m_S x^2\right] = \frac{8\pi^2 m_S x^2}{C} + \frac{4\pi^2}{C}(I_0 + 2I_S)$$

This equation is of the form $y = mx + C$. Therefore, if a graph is plotted between T^2 and x^2, it should be a straight line.

Description of the apparatus: The main aim of this experiment is to show that how the moment of inertia varies with the distribution of mass. The basic relation for this is $I = \Sigma mx^2$.

Two equal weights are symmetrically placed on this system. By varying their positions relative to the axis of rotation, the moment of inertia of the system can be changed.

The Maxwell's needle with two solid cylinders can be used for this purpose. The two weights are symmetrically placed in the tube on either side of the axis of rotation and their positions are noted on the scale engraved by the side of the groove on the hollow X tube as shown in figure 3.24. The time period of the torsional oscillations is now determined. Now the positions of these cylinders are changed in regular steps which cause the variation in distribution of mass. By measuring the time periods in each case, the moment of inertia of the system is studied by the variation in the distribution of mass. For the successful performance of the experiment, the moment of inertia of the suspension system should be much smaller than the moment of inertia of the added weights so that a large difference in the time period may be obtained by varying the position of the added weights. For this purpose a light aluminium channel of about 1.5 metre in length and 5 cm in breadth may be used as shown in Figure (2).

Procedure:

1. As shown in Fig. 3.24(a), put the two solid cylinders symmetrically on either side in the hollow tube of Maxwell's needle and note the distance x of their centre of gravity from the axis of rotation or

   As shown in Fig. 3.24(b), put the two equal weights on the aluminium channel symmetrically on either side of axis of rotation and note the distances x of their centre of gravity from the axis of rotation.
2. Rotate the suspension system slightly in the horizontal plane and then release it gently. The system executes torsional oscillations about the suspension wire.

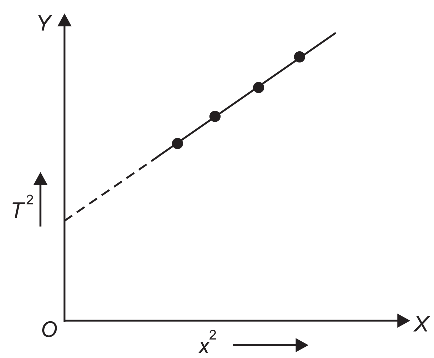


Fig. 3.23

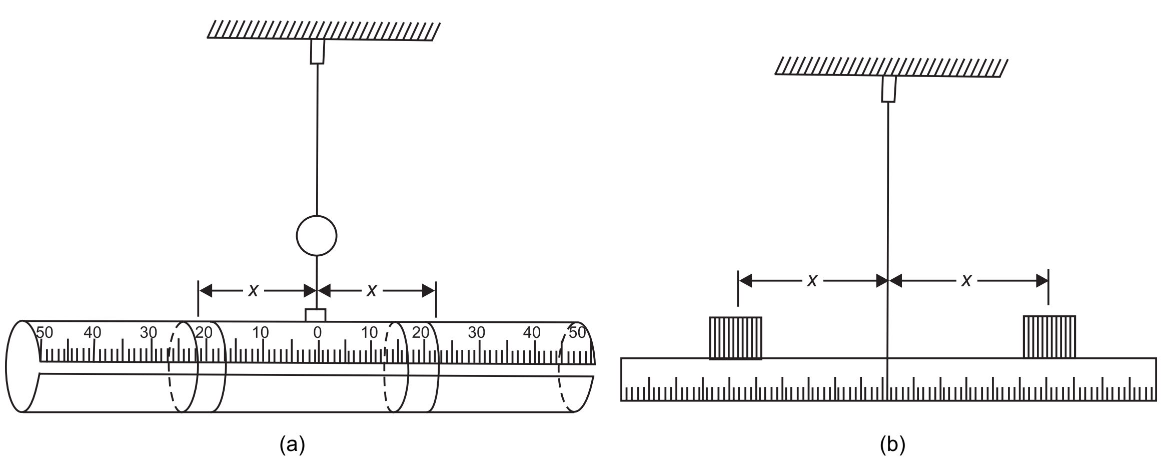


Fig. 3.24

3. Note the time taken by 25–30 oscillations with the help of a stop watch and then divide the total time by the number of oscillations to calculate the time period T.
4. Now displace both the cylinders or added weights by a known distance say 5 cm away from the axis of rotation and determine the time period as discussed above.
5. Take at least 5 or 6 such observations at various values of x by displacing the weights in regular steps of 5 cm.
6. Now plot a graph between x^2 on x-axis and corresponding values of T^2 on y-axis. The graph is shown in figure 3.23.

Observations: Table for time period T and the distance x of the weight:

S. No.	*Distance of the Cylinder of added weight from axis of rotation x meter*	x^2 (m^2)	*Time Period*				T^2 *sec*
			No. of oscillations n	*Time taken t sec*	*Time period T = (t/n) sec*	*Mean time period T, sec*	
1.	x_1	x_1^2	20 25 30	....	....	......	......
2.	x_2	x_2^2	20 25 30				
3.	x_3	x_3^2	20 25 30				
4.	x_4	x_4^2	20 25 30				

Result: Since the graph between T^2 and x^2 comes out to be a straight line, it verifies that the basic theorem $I = \Sigma mx^2$ from which theorem of parallel axes follows, is valid.

Sources of error & precautions:

1. The suspension wire should be free from kinks.
2. The suspension system should always be horizontal.
3. The two solid cylinders or added weights should be identical.
4. Oscillations should be purely rotational.
5. The suspension wire should not be twisted beyond elastic limits.
6. Periodic time should be noted carefully.

3.32 VIVA-VOCE

Q. 1. What do you understand by elasticity?

Ans. The property of the body by virtue of which it regains its original size and shape, when the external forces are removed, is known as elasticity.

Q. 2. What are elastic and plastic bodies?

Ans. Bodies which regain their shape or size or both completely as soon as deforming forces are removed are called perfectly elastic while if completely retain their deformed form is known as perfectly plastic.

Q. 3. What is meant by limit of elasticity?

Ans. If the stress be gradually increased, the strain too increases with it in accordance with Hooke's law until a point is reached at which the linear relationship between the two just ceases and beyond which the strain increases much more rapidly than it is warranted by the law. This value of the stress for which Hooke's law just ceases to be obeyed is called the elastic limit of the material of the body.

Q. 4. What do you mean by stress?

Ans. When a force acts on a body, internal forces opposing the former are developed. This internal force tends to restore the body back to its original form, the restoring or recovering force measured per unit area is called stress. Thus, if F be the force applied normally to an area of cross-section a then stress = F/a.

Q. 5. What do you understand by strain?

Ans. Relative change produced in size or shape or both of body which is subjected to stress is called strain. It is of three types: (*i*) Linear strain, (*ii*) volume strain and (*iii*) Shape strain or Shearing strain.

Q. 6. Explain the all three types of strain.

Ans. (*i*) When a wire is subjected to a tension or compression, the resulting deformation is a change in length and the strain is called linear strain.

(*ii*) If the pressure increments is applied to a body in such a way that the resulting deformation is in volume, without change in shape, the strain is called volume strain.

(*iii*) When tangential stresses act on the faces of the body in such a way that the shape is changed, of course volume remaining the same, the strain is called shearing strain.

Q. 7. How do you differentiate between stress or pressure?

Ans. Though both of them are defined as force per unit area, they carry different meanings. By pressure we mean an external force which necessarily acts normal to the surface, while in stress we take the internal restoring force produced in a body due to the elastic reaction, which do not always act normal to the surface.

Q. 8. What is Hooke's law?

Ans. This law states that within elastic limit, the stress is proportional to strain i.e., stress/strain = a constant, called modulus of elasticity.

Q. 9. How many types of moduli of elasticity do you know?

Ans. There are three types of moduli of elasticity: (*i*) Young's modulus (*ii*) Bulk modulus (*iii*) Modulus of rigidity,

Q. 10. What are the units and dimensions of modulus of elasticity?

Ans. The dimensional formula for modulus of elasticity is $ML^{-1}T^{-2}$ and its units in MKS and CGS systems are newton/metre^2 and dyne/cm^2 respectively.

Q. 11. Define Young's modulus?

Ans. It is defined as the ratio of longitudinal stress to the longitudinal strain within the elastic limits.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

Q. 12. Define Bulk modulus?

Ans. The ratio of normal stress to volume strain within elastic limit is called as Bulk modulus?

$$K = \frac{\text{normal stress}}{\text{volume strain}}$$

Q. 13. Define modulus of rigidity?

Ans. It is defined as the ratio of tangential stress to shearing strain within elastic limits and is denoted by η.

Q. 14. What is compressibility?

Ans. The ratio of volume strain and normal stress is called compressibility $\left[K = \frac{\text{normal stress}}{\text{volume strain}};\ K \text{ is also referred to as incompressibility of the material of the body and } \frac{1}{K} \text{ is called its compressibility}\right]$.

Q. 15. What is the effect of temperature on elastic moduli?

Ans. In general value of elastic moduli decreases with rise in temperature.

Q. 16. Do you know any material whose elasticity increases with decrease of temperature?

Ans. Rubber is such a substance.

Q. 17. Do you know any material whose elasticity is little affected by temperature?

Ans. Some nickel steel alloys *e.g.* elinvar.

Q. 18. How does the elastic limit of a metal change by drawing, hammering and annealing it?

Ans. Drawing and hammering tend to diminish the elastic limit. Annealing tends to increase the elastic limit.

Q. 19. What is the practical use of the knowledge of elastic moduli?

Ans. This enables to calculate the stress and strain that a body of given size can bear. This helps in designing of the body.

Q. 20. How do you explain the meaning of the terms (*i*) limit of proportionality, (*ii*) elastic limit, (*iii*) yield point, (*iv*) breaking stress and (*v*) tensile strength?

Ans.
(*i*) **Limit of proportionality.** Limit upto which extension of wire is proportional to the deforming force. Beyond it, Hooke's law is not obeyed.
(*ii*) **Elastic limit.** The limit beyond which the body does not regain completely its original form even after removal of deforming force. It is very close to limit of proportionality.
(*iii*) **Yield Point.** It is the point beyond which increase in the length is very large even for small increase in the load, and the wire appears to flow.
(*iv*) **Breaking stress.** The maximum stress developed in the wire just before it breaks.
(*v*) **Tensile strength.** Breaking stress for wire of unit cross-section.

Q. 21. What is elastic after effect?

Ans. Some bodies do not regain their original form instantly after the removal of the deforming force, *e.g.*, glass. The delay in recovering the original condition after the deforming force has been withdrawn is known as 'elastic after effect.'

Q. 22. Which substances are exception to above behaviour and to what use are they put?

Ans. Quartz, phosphor-bronze and silver are a few materials which are exception to above behaviour and are extensively used as suspension fibres in many instruments like electrometers, galvanometers, etc.

Q. 23. What precaution is taken in experiments to guard against this error?

Ans. We wait for some time after removal of each load so that body recovers its original conditions completely. For metals not much time is required for this recovery.

Q. 24. What do you understand by elastic fatigue?

Ans. When the wire is vibrating continuously for some days, the rate at which the vibrations die away is much greater than when the wire was fresh. The wire is said to be 'tired' or fatigued' and finds it difficult to vibrate. This is called elastic 'fatigue.'

Q. 25. What is the effect of impurity on elasticity?

Ans. The addition of impurity may increase or decrease the elasticity of material. When a little carbon is added to molten iron, steel is produced which is more elastic than pure iron. While addition of 2% of potassium to gold increases its elasticity many times.

Q. 26. What is elastic hysteresis?

Ans. When a material specimen is subjected to rapid cyclic variations of mechanical strains, the sample is not able to keep pace with the external force. The phenomenon is called elastic hysteresis.

Q. 27. What is Poisson's ratio? What are its units and dimensions?

Ans. Within the elastic limits, the ratio of the lateral strain to the logitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

It has no unit.

Q. 28. Are there any limits to Poissons ratio value?

Ans. Yes, it lies between –1 and $\frac{1}{2}$.

Q. 29. The value of σ is $\frac{1}{2}$ for a material? What inference can you draw about this material?

Ans. It is incompressible.

Q. 30. What is the significance of negative value of Poisson's ratio?

Ans. This means that when a stretching force is applied to a body of that material, it will produce increase in length as well as increase in the perpendicular direction. No known substance shows this behaviour.

Q. 31. Can the method used for determination of Poisson's ratio for rubber be used for glass also?

Ans. No, because in case of glass extension will be extremely small.

Q. 32. What do you mean by a beam?

Ans. A bar of uniform cross-section (circular or rectangular) whose length is much greater as compared to thickness is called a beam.

Q. 33. What are longitudinal filament?

Ans. A rectangular beam may be supposed as made up of a number of thin plane layers parallel to each other. Further, each layer may be considered to be a collection of thin fibres lying parallel to the length of beam. These fibres are called longitudinal filaments.

Q. 34. What is a neutral surface?

Ans. There is a plane in the beam in which the filaments remain unchanged in length when equal and opposite couples are applied at the ends of the beam. The plane is called a neutral plane or neutral surface.

Q. 35. In this experiment, the beam is simply bent, not extended. How is the bending of the beam related, with Young's modulus of the beam?

Ans. When the beam is depressed, it becomes curved. There is elongation of fibres of beam on convex side and contraction on concave side. The longitudinal stress and strain come in picture.

Q. 36. How will the value of *Y* change with a change in length, breadth or thickness of the beam?

Ans. Y remains the same because it is constant for a material.

Q. 37. What do you understand by geometrical moment of inertia?

Ans. Geometrical moment of inertia = $\delta A \cdot K^2$ where δA = area of cross section & K = radius of gyration.

Q. 38. What is bending moment?

Ans. The moment of balancing couple (internal couple) formed by the forces of tension and compression at a section of bent beam is called as bending moment. Bending moment = $\frac{YI_g}{R}$. Here Y = Young's modulus, I_g = geometrical moment of inertia and R = radius of curvature of arc.

Q. 39. Do all the filaments other than those lying in the neutral surface suffer equal change in length?

Ans. The extensions and compressions increase progressively as we proceed away from the axis on either side, so that they are the maximum in the uppermost and the lowermost layers of the beam respectively.

Q. 40. What do you mean by Flexural rigidity?

Ans. The quantity YI is called the 'flexural rigidity'. It measures the resistance of the beam to bending and is quantitatively defined as the external bending moment required to produce unit radius of curvature of the arc into which the neutral axis is bent.

Q. 41. What is a cantilever.

Ans. If the beam be fixed only at one end and loaded at the other, it is called a cantilever.

Q. 42. How does the curvature change along its length?

Ans. Its upper surface becoming slightly convex and the lower one concave.

Q. 43. In the experiment with the beam supported at ends and loaded in the middle, how is the principle of cantilever involved?

Ans. It behaves like a two cantilever.

Q. 44. Why do you measure the length between the two knife edges instead of its full length?

Ans. Before and after the knife edges it is assumed that the portion is inside the clamp so that we apply the principle of cantilever.

Q. 45. Does the weight of the beam not contribute to the depression at the centre? If yes, why have you not taken it into account?

Ans. Yes, it does. This is ignored because in the experiment we calculate the depression due to different weights. Hence the depression due to the weight of the beam is automatically cancelled.

Q. 46. What precaution do you take in placing the beam on knife edges?

Ans. It should be placed in such a way that the knife edges are perpendicular to its length. The beam should rest symmetrically on the knife edges so that equal portions project outside the knife edges. This ensures equal reaction at each knife edge as has been assumed in the theory.

Q. 47. You have kept the beam on the knife edges. Can you keep it with its breadth vertical.

Ans. In that case depression will be very small, because the breadth will now become the thickness of the beam and the depression $\delta \propto \frac{1}{a^3}$.

Q. 48. What is the practical use of this information?

Ans. It is utilized in the construction of girders and rails. Their depth is made much larger as compared to their breadth. These can, therefore, bear much load without bending too much.

Q. 49. Girders are usually of I shape. Why are they not of uniform cross-section?

Ans. In girders filament of upper half are compressed while those of lower half are extended. Since the compression and extension are maximum near the surface, the stresses on the end filaments are also maximum. Hence the top and bottom are thicker than the central portion. This saves a great deal of material without sacrificing the strength of the girder.

Q. 50. Which of the quantities '*b*' or '*d*' should be measured more accurately and why?

Ans. The depth of the bar should be measured very carefully since its magnitude is small and it occurs in the expression of '*y*' in the power of three. An inaccuracy in the measurement of the depth will produce the greatest proportional error in *y*.

Q. 51. What type of beam will you select for your experiment?

Ans. Moderately long and fairly thin.

Q. 52. Why do you select such a beam?

Ans. For two reasons: (*i*) the depression will be large which can be measured more accurately and (*ii*) in bending of beam shearing stress is also produced in addition to the longitudinal one and it produces its own depression. This depression will be negligible only when the beam is long and thin.

Q. 53. How do you ensure that in your experiment the elastic limit is not exceeded?

Ans. The consistency in the readings of depressions both for increasing load and decreasing load indicates that in the experiment the elastic limit is not exceeded.

Q. 54. Does the weight of the bar have any effect?

Ans. The weight of the beam leads to an 'effective load' different from m. However, since the depression due to a load is calculated by subtracting the zero-load readings the weight of the bar does not affect the result.

Q. 55. How do you produce shearing in a rod or wire?

Ans. The upper end of the experimental rod is clamped while the lower end is subjected to a couple. Now each cross-section of rod is twisted about the rod. The angle of twist for any cross-section of the rod being proportional to its distance from fixed end. Thus the material particles of the rod are relatively displaced with respect to the particles in adjoining layer and the rod is sheared.

Q. 56. What is the angle of twist and how does this angle vary?

Ans. When one end of the rod is clamped and a couple is applied at other end, each circular cross section is rotated about the axis of the rod through certain angle. This angle is called the angle of shear.

Q. 57. What is the difference between the angle of twist and the angle of shear? How are these angles related?

Ans. $\phi = \frac{x\theta}{l}$, where l is the length of the rod and x being the radius of co-axial cylinder under consideration.

Q. 58. What are the values of twisting couple and restoring couple?

Ans. Twisting couple is equal to Mgd, where M = mass placed at each pan and 'd' being the diameter of cylinder.

Restoring couple = $\frac{\pi\eta r^4}{2l}\theta$.

Q. 59. What is the condition of equilibrium in 'η' experiment?

Ans. Mgd = $\frac{\pi\eta r^4}{2l}\theta$

Q. 60. What do you mean by torsional rigidity of a wire?

Ans. This is defined as restoring couple per unit radian twist *i.e.*,

$$C = \frac{\pi\eta r^4}{2l}$$

Q. 61. Do you prefer to use an apparatus provided with a cylinder of larger or smaller radius?

Ans. We shall prefer to take a cylinder of larger radius because twisting couple will be greater.

Q. 62. On what factors does the twist produced in the wire for a given twisting couple depend?

Ans. It depends upon the torsional rigidity of the wire. Smaller the rigidity, greater is the twist and vice-versa. Now $c = \frac{\pi\eta r^4}{2l}$. Hence twist will depend upon length, radius and material of the wire.

Q. 63. Why it is called a statical method?

Ans. This is called a statical method because all observations are taken when all parts of apparatus are stationary.

Q. 64. In vertical apparatus why are levelling screws provided at the base?

Ans. These screws are used to adjust the apparatus such that the experimental wire passes through the centres of the graduated scales to avoid the error due to eccentricity.

Q. 65. Why do you read both the ends of the pointer?

Ans. This eliminates the error due to any eccentricity left even after above adjustment.

Q. 66. Why is the value of η for a thinner wire slightly higher than that for a thicker wire of the same material?

Ans. The wires are drawn by squeezing the molten metal through holes (dies), hence the outer layers are necessarily tougher than the inner ones. Therefore the value of η for a thinner wire is slightly higher than for a thicker wire of the same material.

Q. 67. Why do you measure the radius of wire so accurately?

Ans. Because it occur in fourth power in the formula.

Q. 68. Will the value of modulus of rigidity be the same for thin and thick wires of the same material?

Ans. For a given material, the value should be the same. But we know that wires are made by drawing the metal through a hole. The outer surfaces become harder than inner core. Thus the rigidity of a fine wire will be greater than thick wire.

Q. 69. When the modulus of rigidity can be determined by statical method, what is the necessity of dynamical method.

Ans. The statical method is suitable for thick rods while dynamical method is suitable for thin wires.

Q. 70. What is a torsional pendulum?

Ans. A body suspended from a rigid support by means of a long and thin elastic wire is called torsional pendulum.

Q. 71. Why it is called a torsional pendulum?

Ans. As it performs torsional oscillations, hence it is called a torsional pendulum.

Q. 72. What are various relationship between elastic constants?

Ans. $Y = 2\eta(1+\sigma)$, $Y = 3k\,(1-2\sigma)$

$$\sigma = \frac{3k - 2\eta}{6k + 2\eta},\ Y = \frac{9\eta k}{\eta + 3k}$$

Q. 73. What is the unit of Poisson's ratio?

Ans. It has no unit because it is a ratio.

Q. 74. When do you measure the diameter of the tube?

Ans. We measure the diameter of tube at no load position.

Q. 75. Can you not calculate the change in volume by knowing the longitudinal extension?

Ans. No, the contraction in diameter should also be known.

Q. 76. If a graph is plotted between change in volume and change in length, what kind of curve will you get?

Ans. It gives a straight line.

Q. 77. Can you use this method for determining σ for glass?

Ans. No, in case of glass the extension produced is so small that it can not be measured.

Q. 78. Which method do you suggest for glass?

Ans. Cornu's method.

Q. 79. What is a flywheel?

Ans. It is a large size heavy wheel mounted on a long axle supported on ball bearing.

Q. 80. Why the mass of a flywheel is concentrated at rim?

Ans. This increases the radius of gyration and hence the moment of inertia of the flywheel.

Q. 81. What is the practical utility of a flywheel?

Ans. It is used in stationary engines to ensure a uniform motion of the machine coupled to the engine.

Q. 82. Is flywheel used in mobile engines also?

Ans. No, it is not needed in mobile engines, because the heavy body of vehicle itself serves the purpose of flywheel.

Q. 83. How and why does the flywheel start rotating?

Ans. When a weight is hanged near the axle, it has certain amount of potential energy. After realising it, its potential energy is converted into kinetic energy of its own motion, into kinetic energy of rotating of flywheel and in overcoming the force of friction between the axle and ball bearings.

$$\therefore \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 2\pi n_1 F.$$

Q. 84. Why the flywheel continues its revolutions even after the cord has slipped off the axle.

Ans. It continues its revolutions due to its large moment of inertia.

Q. 85. Then, why does it stop after a very short time?

Ans. The energy of the flywheel is dissipated in overcoming the friction at the ball bearings.

Q. 86. What is the purpose of finding n_2?

Ans. The friction offered by the ball bearings is very small but it is not negligible. To account for the work done by weight against friction, the number of revolutions made by flywheel after the weight is detached should be found.

Q. 87. Can you use a thin wire instead of a string?

Ans. No, we cannot use a wire because metals are pliable and so when the wire unwinds itself, some amount of work will also be done in straightening the wire.

Q. 88. Why do you keep the loop slipped over the peg loose?

Ans. We keep the loop slipped over the peg loose so that it may get detached as soon as the string unwinds itself and does not rewind in opposite direction.

Q. 89. What is the harm if the thread overlaps in winding round the axle.

Ans. In this case the couple acting on the wheel will not be uniform and hence the flywheel will not rotate with uniform acceleration.

Q. 90. What is a spiral spring?

Ans. A long metallic wire in the form of a regular helix of given radius is called a spiral spring.

Q. 91. What types of springs do you know?

Ans. There are two types of springs: (*i*) Flat and (*ii*) non-flat. When the plane of the wire is perpendicular to the axis of the cylinder, it is flat and when the plane of wire makes certain angle with the axis of cylinder, it is non flat.

Q. 92. What is the effective mass of a spring?

Ans. In calculations total energy of the spring, we have a quantity $\left(M + \frac{m}{3}\right)$ where M is the mass suspended and m, the mass of the spring. The factor $\left(\frac{m}{3}\right)$ is called the effective mass of the spring.

Q. 93. What do you understand by restoring force per unit extension of a spiral spring?

Ans. This is defined as the elastic reaction produced in the spring per unit extension which tends to restore it back to its initial conditions.

Q. 94. How does the restoring force change with length and radius of spiral spring?

Ans. This is inversely proportional to the total length of wire and inversely proportional to the square of radius of coil.

Q. 95. How the knowledge of restoring force per unit extension is of practical value?

Ans. By the knowledge of restoring force per unit extension, we can calculate the correct mass and size of the spring when it is subjected to a particular force.

Q. 96. How are Y and η involved in this method?

Ans. First of all the wire is placed horizontally between two bars. When the bars are allowed to vibrate, the experimental wire bent into an arc. Thus the outer filaments, are elongated while inner ones are contracted. In this way, Y comes into play. Secondly, when one bar oscillates like a torsional pendulum, the experimental wire is twisted and η comes into play.

Q. 97. Is the nature of vibrations the same in the second part of the experiment as in the first part?

Ans. No, in the second case, the vibrations are torsional vibrations.

Q. 98. Should the moment of inertia of the two bars be exactly equal?

Ans. Yes, if the two bars are of different moment of inertia, then their mean value should be used.

Q. 99. Do you prefer to use heavier or lighter bars in this experiment?

Ans. We shall prefer heavier bars because they have large moment of inertia. This increases the time period.

Q. 100. Can you not use thin wires in place of threads?

Ans. No, because during oscillations of two bars, the wires will also be twisted and their torsional reaction will affect the result.

Q. 101. From which place to which place do you measure the length of wire and why?

Ans. We measure the length of the wire from centre of gravity of one bar to the centre of gravity of the other because it is length of the wire which is bent or twisted.

Q. 102. Is there any restriction on the amplitude of vibration in both part of experiments?

Ans. When the two rods vibrate together, the amplitude of vibration should be small so that the supporting threads remain vertical and there is no horizontal component of tension in the threads. In case of torsional oscillations there is no restriction on the amplitude of oscillations but the wires should not be twisted beyond elastic limits.

Q. 103. Why do the bars begin oscillating when the thread tied to them is burnt? Do they perform S.H.M.?

Ans. When the wire is bent into circular arc and the thread is burnt, the wire tries to come back to its original position due to elastic reaction. In doing so it acquires kinetic energy. Due to this energy the wire overshoots the initial position and becomes curved in another direction. The process is repeated and the bar begins to oscillate. Yes, the rod performs simple harmonic motion.

Q. 104. What do you mean by Poisson's ratio?

Ans. Within the elastic limits, the ratio of the lateral strain to the longitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

Q. 105. What is the principle of Maxwell needle experiment?

Ans. When the Maxwell's needle is given a small angular displacement and released, it starts oscillating. The periodic time of the needle is related with the elasticity of the wire.

Q. 106. On what factors does the periodic time depend?

Ans. It depends upon (*i*) moment of inertia of the needle about wire, and (*ii*) length, radius and material of wire.

Q. 107. What would be the change in periodic time when (*i*) *I* is doubled, (*ii*) *l* is doubled and (*iii*) *r* is doubled.

Ans. We know that $T = 2\pi\sqrt{\frac{I}{C}}$ where $C = \frac{\pi\eta r^4}{2l}$, hence

(*i*) T increases to $T\sqrt{2}$

(*ii*) T increases to $T\sqrt{2}$

(*iii*) T decreases to $T/4$.

Q. 108. Does the time period change, by changing the position of hollow and solid cylinders? Why.

Ans. Yes, the distribution of mass is changed about the axis of rotation and hence the moment of inertia of the needle is changed.

Q. 109. When you change neither the mass of the needle nor the axis of rotation, why does the period change?

Ans. We known that time period depends upon the moment of inertia of oscillating body while the moment of inertia depends upon the distribution of mass besides the total mass and axis of rotation.

Q. 110. Will you prefer to use a long and thin wire or a short and thin wire?

Ans. We shall prefer to use a long and thin wire so that C will be small and periodic time will be greater.

Q. 111. Will the value of modulus of rigidity be the same for thin and thick wires of the same material?

Ans. For a given material, the value should be the same. But we know that wires are made by drawing the metal through a hole. The outer surface becomes harder than inner core. Thus the rigidity of a fine wire will be greater than thick wire.

Q. 112. When the modulus of rigidity can be determined by statical method, what is the necessity of dynamical method?

Ans. The statical method is suitable for thick rods while dynamical method is suitable for thin wires.

Q. 113. How the moment of inertia of a system can be changed?

Ans. The moment of inertia of a system can be changed by varying the distribution of mass.

Q. 114. Which apparatus you are using for this purpose?

Ans. We are using Maxwell's needle for this purpose.

Q. 115. How do you vary the distribution of mass here?

Ans. By changing the positions of two weights symmetrically inside the tube.

Q. 116. What will be the effect on time period of the system by varying the distribution of mass?

Ans. The time period T increases as x increases.

Q. 117. Can you verify the theorem of parallel axes with this experiment?

Ans. Yes, if a graph is plotted between T^2 and x^2, it comes out to be a straight line. This verifies the theorem of parallel axes.

Q. 118. What type of motion is performed by the needle?

Ans. The needle performs the simple harmonic motion.

Q. 119. Should the amplitude of vibration be small here?

Ans. It is not necessary because the couple due to torsional reaction is proportional to the angle of twist. Of course, the amplitude should not be so large that the elastic limit is crossed as the wire is thin and long.

EXERCISE

Q. 1. What do you mean by shearing of body?
Q. 2. How many types of stresses do you know? How will you produce these stresses?
Q. 3. How do you measure the depression at the middle point of the beam?
Q. 4. Why do you load and unload the beam in small steps and gently?
Q. 5. What is meant by modulus of rigidity?
Q. 6. What are shearing stress and strains and what are their units and dimensions?
Q. 7. What is the nature of stress in the case of 'η'.
Q. 8. How and where do you apply tangential stress in this case?
Q. 9. What is the principal of the statical experiment?
Q. 10. What is the value of the restoring couple?
Q. 11. Why do you take readings with increasing and decreasing couples?
Q. 12. What is Poisson's ratio?
Q. 13. What is the value of σ for homogeneous and isotropic materials.
Q. 14. How can you find out the Poisson's ratio for rubber.
Q. 15. Why should you wait for about 5 min, after each addition or removal of a load before taking observation.
Q. 16. Is there any limit to the load placed on the hanger.
Q. 17. Why should there be no air bubble inside the tube?
Q. 18. Why should the spiral spring be suspended exactly vertically?
Q. 19. Why should the extension of the string be small.
Q. 20. Why should the amplitude of oscillation of the spring be small?
Q. 21. Do you get the same value of restoring force per unit extension of the spring from the statical and dynamical experiments. If not, why?

Q. 22. How can you determine the mass of the spring?
Q. 23. What is Maxwell's needle?
Q. 24. How do you measure η with Maxwell's needle?
Q. 25. Which is better: an ordinary pointer or a telescope and scale method of observing oscillations?
Q. 26. Can you improve the pointer method to be nearly as good?
Q. 27. What is parallax and how can you best remove it?
Q. 28. Is it necessary that the oscillations should have small amplitude, if not, why?
Q. 29. Why do you measure the diameter so accurately?
Q. 30. Why should the needle be kept horizontal throughout the experiment?
Q. 31. Why does the needle oscillate when released after twisting the wire?
Q. 32. Which is better, statical or dynamical method?
Q. 33. Do you get the same value for η from the statical and dynamical methods? If not, why?