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Department of Physics

Class :- S.Y.B.Sc. ***Physics Paper-I Semester-I***

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QUESTION BANK

A.] Short Answer Type Questions:-

1.]Complex numbers-

1) Define complex number and its complex conjugate.
2) If z= x+iy then what is the modulus of z?
1) When two complex numbers are said to be equal?
2) What is Argand diagram?
3) State De Moivre's theorem.
4) Using Eular's formula, show that
   i. Cos θ = $\frac{e^{i\theta}+e^{-i\theta}}{2}$
5) If z= $i + i^2$, find x and y.

2.]Partial Differentiation-

1) Define partial Differential equation.
2) What is meant by an implicit function?
3) Write conditions for maxima and minima for one variable.
4) What do you mean by explicit function?

3.]Vector Algebra-

1) What is a null vector?
2) What is meant by the position vector of a point?
3) What is geometrical significance of scalar product of two vectors?
4) Define: (i) Scalar triple product. (ii) Vector triple product. Show that the magnitude of vector product $\overrightarrow{A}$ X $\vec{B}$ gives the area of parallelogram.

4.]Vector Analysis-

1) How are (i) scalar field and (ii) vector field represented?
2) Define the gradient of a scalar field.
3) Define the divergence of a vector field.
4) What do you mean by a solenoidal vector field? Give one example.

5) Define Laplacian operator. Is it scalar or vector operator?

5.]Differential Equation-

1) What is ordinary differential equation? State any three differential equations.
2) What is partial differential equation? State any three frequently occurring partial differential equations in physics.
3) Explain the following terms: (i) Linearity, (ii) Homogeneity.
4) Discuss singularity at x=∞.
5) **B.] Long Answer Type Questions:**

1.]Complex numbers-

1) What is an Argand diagram? Explain addition of two complex numbers by using an Argand diagram.
2) Explain subtraction of two complex numbers by using an Argand diagram.
3) Explain multiplication and division of two complex numbers by using an Argand diagram.
4) Using the expressions cosθ= $\frac{(e^{i\theta}+e^{-i\theta})}{2}$ and sinθ= $\frac{e^{i\theta}+e^{-i\theta}}{2i}$

   Prove that: (i) sin2θ = 2sinθcosθ and (ii) cos2θ = $\cos^2\theta - \sin^2\theta$
5) Define hyberbolic functions and show that

$$\cosh^2 z - \sinh^2 z = 1$$

2.]Partial Differentiation-

1) Distinguish between implicit and explicit function.
2) If f(x,y,z)=0, show that (i) $\left(\frac{\delta y}{\delta y}\right)_z \left(\frac{\delta y}{\delta x}\right)_z$ = 1,(ii) $\left(\frac{\delta y}{\delta z}\right)_x \left(\frac{\delta z}{\delta x}\right)_y \left(\frac{\delta x}{\delta y}\right)_z$ = -1
3) If F= f(x,y) and x=x(u) and y= y(u) then show that:

   $\frac{dF}{du} = \frac{\delta F}{\delta x}\frac{dx}{du} + \frac{\delta F}{\delta y}\frac{dy}{du}$
4) Discuss conditions for maxima and minima for one variable and many variables.
5) If F= f(x, y), x= r cos θ and y= r sin θ then prove the equations $\left(\frac{\delta F}{\delta x}\right)^2 + \left(\frac{\delta F}{\delta y}\right)^2 = \left(\frac{\delta F}{\delta r}\right)^2 + \frac{1}{r}\left(\frac{\delta F}{\delta \theta}\right)^2$

3.]Vector Algebra-

1) What is a scalar? Show that: $\vec{A}.\left(\vec{B}X\vec{C}\right) = \vec{B}.\left(\vec{C}X\vec{A}\right) = \vec{C}\ .\left(\vec{A}X\vec{B}\right)$
2) Show that vector triple product $\vec{A}X\left(\vec{B}X\vec{C}\right) = \vec{B}.\left(\vec{A}.\vec{C}\right) - \vec{C}\left(\vec{A}.\vec{B}\right)$ What is the vector triple product?

3) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a triangle, show that the vector area of the triangle is $\frac{1}{2}\left[\vec{b}X\vec{c} + \vec{c}X\vec{a} + \vec{a}X\vec{b}\right]$

4.]Vector Analysis-

1) Explain the physical significance of a gradient.
2) Explain the statement : "the gradient of a scalar field is invariant".
3) Express curl of $\vec{V}$ in terms of Cartesian components.
4) Explain the statement "curl is known as rotational or rot".

5.]Differential Equation-

1) Explain how will you determine the point $x = x_0$ is regular singular point of the given linear second order homogenous differential equation.
2) The point a infinity requires a special consideration to decide its singularity structure for a given differential equation. Discuss.

C.] Unsolved Problems:

1.]Complex numbers-

1) Prove the relations: (i) sinh (iθ)= i sinθ (ii) cosh iθ = cos θ (iii) sinh θ = -i sin (iθ) (iv) cosh θ = cos (iθ)
2) If z = 1 + $\sqrt[3]{3i}$, determine (i) $|z|$, (ii) Arg (z), (iii) $\bar{z}$, (iv) Re (z) and (v) Im (z). (**Ans.** $2, 30^o, 1 - \sqrt{3}$i, 1, $\sqrt{3}$i)
3) If x + iy = $\frac{1+\sqrt{5}i}{1-\sqrt{5}i}$, determine $|z|$. (**Ans.**$\frac{9}{34}$, $\frac{19}{34}$)
4) If $z = \frac{1+\sqrt{5}i}{1-\sqrt{5}i}$, determine $|z|$. (**Ans.** 1)
5) Determine the value of $\left(cos\frac{\pi}{4} + i\ sin\frac{\pi}{4}\right)\left(cos\frac{3\pi}{8} + isin\frac{3\pi}{8}\right)^2$ (**Ans.** -1)
6) Express the complex number –πi into (i) polar form and (ii) exponential form.(**Ans.**$-\pi\left(\cos\frac{\pi}{2} + isin\frac{\pi}{2}\right), -\pi e^{\frac{i\pi}{2}}$)
7) Interpret the following equation geometrically. $|z - 3| = \sqrt{2}|z - 4|$where z = x+ iy (**Ans.** Circle with origin (5,0) and radius $\sqrt{2}$)
8) Simplify $\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^6 - \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^6$(**Ans.** 0)
9) Determine the value of $(1 + i)^8$.(**Ans.** 16)
10) Simplify and show that the given sum is a rational number $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$.(**Ans.**$\frac{-8}{29}$)
11) Determine x and y if$x + iy = \left(\frac{1+i}{1-i}\right)^4$(**Ans.**$cos2\pi, sin2\pi$)

11) Prove: (i) $\cos(\alpha+\beta i) = cos\alpha\ cosh\beta - i\ sin\alpha\ sinh\beta$

(ii) $\sinh(\alpha+\beta i) = sinh\alpha\ cos\beta + i\ cosh\alpha\ sin\beta$

12) Transform $\frac{1}{(1-i)^2}$ into the trigonometrical form.(**Ans.** $cos\frac{\pi}{2} + i\ sin\frac{\pi}{2}$)

13) Express $\left(\frac{9-7i}{2-3i}\right)$ in the form of x + iy.(**Ans.** 3+i)

2.]Partial Differentiation-

1) If $F = x^2y + xy^2 - axy$, find F_{xx} , F_{yy}, F_{xy} and F_{yx}.
2) Show that $\frac{\delta^2 F}{\delta x^2} + \frac{\delta^2 F}{\delta y^2} + \frac{\delta^2 F}{\delta z^2}$ = 0, where F = $\frac{1}{\sqrt{x^2+y^2+z^2}}$.
3) For u = $e^x \cos y$, verify that $\frac{\delta^2 u}{\delta x \delta y} = \frac{\delta^2 u}{\delta y \delta x}$.
4) If $E = E_0 \sin(\omega t - kx)$ where E_0, ω and k are constants, show that $\frac{\delta^2 E}{\delta t} = \frac{\omega^2}{k^2}\frac{\delta^2 E}{\delta x^2}$
5) If F (x,y) = $ln\left[\frac{xy}{x^2+y^2}\right]$, find $\frac{\delta F}{\delta x}$ and $\frac{\delta F}{\delta y}$.(**Ans.** $\frac{y^3-x^2y}{xy(x^2+y^2)}$; $\frac{x^3-xy^2}{xy(x^2+y^2)}$)
6) Show that following equations are exact differential:

   (i) $[2x + y\cos(xy)]dx + x\cos(xy)dy = 0$

   (ii) $3x\,(xy-2)dx + (x^2+2y)dy = 0$.
7) The acceleration of gravity can be found from the length *l* and period T of pendulum using formula $g = \frac{4\pi^2 l}{T^2}$. Find the relative error in g in the worst case if the relative error in *l* is 5% and relative error in T is 2%. (**Ans.** 9%)
8) Find the approximate value of $\sqrt{(2.99)^2 + (3.99)^2}$ using method of differentials. (**Ans.** 4.986)
9) Using the method of differential equation, find the approximate value of $[(6.01)^2 + (8.01)^2]^{1/2}$. (**Ans.** 10.014)

10)The resistance R of a uniform wire of length *l* and resistance R is given by $R = \frac{\sigma l}{\pi r^2}$ where σ is the specific resistance. If error in the measurement of length and radius are 2% and 3% respectively, find maximum possible percentage error in resistance. (**Ans.** 8%)

11)If $u = \sin\frac{x}{y}$ and $x = e^t, y = t^2$ verify $\frac{du}{dt} = \frac{\delta u}{\delta x}.\frac{dx}{dt} + \frac{\delta u}{\delta y}.\frac{dy}{dt}$

12)A wooden cylinder of radius 7 cm and height 10 cm is to be coated with a thin silver sheet of thickness 0.1 cm. Find volume of silver sheet. [**Hint:** $v = \pi r^2 h, dv = 2\pi rhdr + \pi r^2 dh, dr = 0.1\ cm, dh = 0.2cm.$] (**Ans.** 74.8 cm^3)

13) Find the slope of the tangent to the curve $x^3 + xy + y^2 - 4 = 0$ at $x = 2, y = -2$. (**Ans.** Slope= 1)

14) Find the position and nature of the stationary points of the function $f(x) = 2x^3 - 3x^2 - 36x + 2$. (**Ans.** Min.at x= 3, max.at x= -2)

15) Find the position and nature of the stationary points for the following function $f(x) = x^3 - 3x^2 + 3x$. (**Ans.** Inflection at x=1)

3.]Vector Algebra-

1) Prove that $\vec{A} = \vec{\imath} + 2\vec{\jmath} + 8\vec{k}$ and $\vec{B} = 2\vec{\imath} + 3\vec{\jmath} - \vec{k}$ are perpendicular to each other. (**Ans.**$\vec{A}.\vec{B} = 0$)
2) Find the value of p which makes vectors $\vec{A} = p\vec{\imath} + 2\vec{\jmath} + 3\vec{k}$ and $\vec{B} = -\vec{\imath} + 5\vec{\jmath} + p\vec{k}$ perpendicular. (**Ans.** p= -5)
3) If $\vec{A} = 4\vec{\imath} - \vec{\jmath} + 3\vec{k}$ and $\vec{B} = -2\vec{\imath} + \vec{\jmath} - 2\vec{k}$, find a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$. (**Ans.**$\pm\frac{(\vec{\imath}-2\vec{\jmath}-3\vec{k})}{3}$)
4) Find the work done in moving an object along a straight line from (3,2,-1) to (2,-1,4) in a force field given by $\vec{F} = 4\vec{\imath} - 3\vec{\jmath} + 2\vec{k}$. (**Ans.** 15)
5) Find the area of a triangle formed by the points whose position vectors are $3\vec{\imath} + \vec{\jmath}, 5\vec{\imath} + 2\vec{\jmath} + \vec{k}, \vec{\imath} - 2\vec{\jmath} + 3\vec{k}$. (**Ans.**$\sqrt{29}$)
6) If $\vec{A} = 3\vec{\imath} - \vec{\jmath} - 2\vec{k}$ and $\vec{B} = 2\vec{\imath} + 3\vec{\jmath} + 3\vec{k}$, find: (a) $|\vec{A}X\vec{B}|$, (b)$(\vec{A} + 2\vec{B})X(2\vec{A} - \vec{B})$, (c) $|(\vec{A} + \vec{B})X(\vec{A} - \vec{B})|$. (**Ans.** (a) $\sqrt{195}$, (b) $-25\vec{\imath} + 35\vec{\jmath} - 55\vec{k}$, (c) $2\sqrt{195}$)
7) A force given by $\vec{F} = 3\vec{\imath} + 2\vec{\jmath} - 4\vec{k}$ is applied at point (1,-1,2). Find the moment of force about the point (2,-1,3). (**Ans.**$2\vec{\imath} - 7\vec{\jmath} - 2\vec{k}$)
8) Find the volume of parallelepiped whose edges are represented by $\vec{A} = 2\vec{\imath} - 3\vec{\jmath} + 4\vec{k}, \vec{B} = \vec{\imath} + 2\vec{\jmath} - \vec{k}, \vec{C} = 3\vec{\imath} - \vec{\jmath} + 2\vec{k}$. (**Ans.** 7)
9) If $\vec{A} = \vec{\imath} + \vec{\jmath} - \vec{k}, \vec{B} = \vec{\imath} - \vec{\jmath} + \vec{k}, \vec{C} = \vec{\imath} - \vec{\jmath} - \vec{k}$, find the vector $\vec{A}X(\vec{B}X\vec{C})$. (**Ans.**$2\vec{\imath} - 2\vec{\jmath}$)
10) Angular velocity of a rotating rigid body about an axis of rotation is given by $\vec{\omega} = 4\vec{\imath} + \vec{\jmath} - 2\vec{k}$. Find the linear velocity of a point P on the body whose position vector relative to a point on the axis of rotation is $2\vec{\imath} - 3\vec{\jmath} + \vec{k}$. (**Hint:** $\vec{v} = \vec{\omega}X\vec{r}$) (**Ans.**$-5\vec{\imath} - 8\vec{\jmath} - 14\vec{k}, \sqrt{285}$)
11) If $\vec{A}X\vec{B} = \vec{C}X\vec{D}$ and $\vec{A}X\vec{C} = \vec{B}X\vec{D}$, show that $\vec{A} - \vec{D}$ and $\vec{B} - \vec{C}$are parallel vectors.
12) Prove : $(\vec{A}X\vec{B}).(\vec{C}X\vec{D}) + (\vec{B}X\vec{C}).(\vec{A}X\vec{D}) + (\vec{C}X\vec{A}).(\vec{B}X\vec{D}) = 0$.
13) Show that vectors $\vec{P} = 4\vec{\imath} - 3\vec{\jmath} + 5\vec{k}, \vec{Q} = -3\vec{\imath} + 2\vec{\jmath} + 4\vec{k}$ and $\vec{R} = 2\vec{\imath} - \vec{\jmath} - \vec{k}$ and not coplanar.

14) Find the volume of a parallelepiped whose three coterminous are described by vectors $\vec{\imath} + 2\vec{\jmath}, 4\vec{\jmath}$ and $\vec{\jmath} + 3\vec{k}$.

4.]Vector Analysis-

1) If $\Phi(x, y, z) = 3xy + yz$, determine $\vec{\nabla}\Phi$ at point (1,1,1). (**Ans.**$3\vec{\imath} + 4\vec{\jmath} + \vec{k}$)
2) If $\Phi = 2xz^4 - x^2y$, find $\vec{\nabla}\Phi$ and $|\vec{\nabla}\Phi|$ at point (2,-2,-1). (**Ans.**$10\vec{\imath} - 4\vec{\jmath} - 16\vec{k}; 2\sqrt{93}$)
3) Determine a unit normal to the surface $x^2 + y^2 + z^2 = 4$ at point (1,-2,3). (**Ans.** $\frac{\mathbf{2}}{\sqrt{\mathbf{56}}}\vec{\imath} - \frac{4}{\sqrt{56}}\vec{\jmath} + \frac{6}{\sqrt{56}}\vec{k}$)
4) Find the directional derivative of $\Phi = 2xz^2 - xy^3$ at point (1,1,-2) in the direction $2\vec{\imath} - \vec{\jmath} + 2\vec{k}$. (**Ans.** 1/3)
5) If $\vec{V} = y^2z\vec{\imath} + xz\vec{\jmath} + 3z\vec{k}$, find $\vec{\nabla} \cdot \vec{V}$. (**Ans.** 3)
6) Prove that the vector $\vec{A} = 3y^4z^2\vec{\imath} + 4x^2z^2\vec{\jmath} - 3x^2y^2\vec{k}$ is a solenoid. (**Hint.**$\vec{\nabla} \cdot \vec{A} = 0$)
7) Show that : $\vec{A} = (2x^2 + 8xy^2z)\vec{\imath} + (3x^3y - 3xy)\vec{\jmath} - (4y^2z^2 + 2x^3z)\vec{k}$ is not solenoidal but $\vec{B} = xyz^2\vec{A}$ is solenoidal.
8) Determine the constant 'p' such that the vector:$\vec{V} = (x + y)\vec{\imath} + (y - z)\vec{\jmath} + (x + 2pz)\vec{k}$is solenoidal. (**Ans.** p=-1)
9) Prove that the vector: $\vec{A} = (4xy - z^2)\vec{\imath} + 2x^2\vec{\jmath} - 3xz^2\vec{k}$is irrotational. (**Hint.**$\vec{\nabla} \cdot \vec{A} = 0$)
10) If $\vec{A}$ and $\vec{B}$are irrotational, show that $\vec{A}X\vec{B}$ is solenoidal.
11) If $\vec{A}$ is irrotational show that$\vec{A}X\vec{r}$ is solenoidal, where $\vec{r}$ is a position vector.
12) Show that: $\vec{\nabla}\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$
13) Show that: $\vec{\nabla}\left(\frac{\vec{r}}{r^3}\right) = 0.$
14) Determine the work done in moving a particle in a force field given by $\vec{F} = 7xy\vec{\imath} + 2z\vec{\jmath} + x\vec{k}$ along the curve $x = 2t^2, y = t, z = t^2 - 3t$ from t=0 to t=1. (**Ans.**$\frac{118}{15}$J)

5.]Differential Equation-

1) State the condition under which a given differential equation is asaid to be (i) linear, (ii) homogeneous. State whether each of the following differential equations is linear or non-linear, homogeneous or non-homogenous and what is the order of each equation?
   (a) $xy'' + (3\sin^2 x)y' = 5\,cosxy$(**Ans.** Linear, homogeneous, II order) (b) $y''' - 5y' = 3x$(**Ans.**Linear, inhomogeneous, III order) (c) $y'' - yy' + 7y = 3$ (**Ans.**Non-Linear, inhomogeneous, II order) (d) $x^2y'' + e^xy' + (x^2 - 1)y = 0$ (**Ans.**Linear, homogeneous, II order)
2) Decide degree and order of each of the following differential equations:

(i) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ (**Ans.** Degree=1, Order=2)

(ii) $\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = x$ (**Ans.** Degree=1, Order=2)

(iii) $\frac{d^3y}{dx^3} + 6\sqrt{\left(\frac{dy}{dx}\right)^2 + y^2} = 0$ (**Ans.** Degree=2, Order=3)

3) Show that $x = 0$ is a regular singular point of the differential equation $x^2y'' - x(2-x)y' + (2+x^2)y = 0$.
4) Show that $x = 0$ is a ordinary point of the differential equation $y'' + xy' + (x^2+2)y = 0$.
5) Show that $x = 0$ is a regular singular point of the differential equation $x(x-1)y'' + (3x-1)y' + y = 0$.
6) Show that the points $x = 1, -1$ are the regular singular singularities of the associated Legendre equation$(1-x^2)y'' - 2xy' + \left[l(l+1) - \frac{m^2}{1-x^2}\right]y = 0$.
7) Show that the point $x = 1$ is a regular singular point of the Legendre differential equation $(1-x^2)y'' - 2xy' + l(l+1)y = 0$.
8) Show that the point $x = 0$ is a regular singular point of the Leguerre's differential equation $xy'' + (1-x)y' + \lambda y = 0$.
9) Show that the point $x = 0$ is an ordinary point of the differential equation $(1+x^2)y'' + x\frac{dy}{dx} - y = 0$.